Teachers in transition: Moving towards CAS-supported classrooms

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Abstract: In the near future many teachers may be required to incorporate CAS into their teaching practices. Based on classroom observations and interviews over two years, this paper reports how two teachers made the transition from using graphics calculators to CAS calculators while teaching differential calculus to upper secondary school students. Both teachers taught with CAS in ways that were consistent with their beliefs about learning and teaching. Over two years, the teachers' teaching approach and purpose for use of technology were stable and seemed to be underpinned by their beliefs about learning. In contrast, both teachers made changes to the content they taught (and thus what they used technology for) in response to new institutional knowledge. Content choice seemed to be underpinned by the teachers' purpose for teaching. Two other influences impacted on what the teachers taught and how they taught it: the teachers' own content knowledge, and the lack of legitimacy of CAS as a tool for learning and assessment in the trial school and wider educational community. The extent of differences noted between the responses of just two teachers indicates that there will be many responses to using CAS in classrooms, as teachers aim to achieve different learning goals and interpret their responsibilities to students in different ways.

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ZDM-Classification:

1 Teaching with technology in the mathematics classroom

It is generally acknowledged that classroom teaching practices are influenced by teachers' underlying beliefs and knowledge about mathematics and mathematics teaching. Reporting on a professional development project during which teachers explored learning and teaching with computer activities, Noss and Hoyles (1996) monitored changes in the ways that the teachers used Microworlds technology. These changes served as a ″window″ on the teachers' mathematical beliefs and pedagogy. Noss and Hoyles observed that ″there is a mutually constructive relationship between what the teachers believe and what they do″(p.201). They maintained that while changing their pedagogy ″[t]eachers are not pushed arbitrarily by 'constraints'. Neither are they free agents″(p.201). This paper provides a detailed description of how two teachers adapted to teaching with CAS and exposes their underlying different dispositions to teaching (as in the Noss and Hoyles study that involved the introduction of a new technology). It shows that their teaching practices (and the changes they made) were influenced by their beliefs about learning and purpose for teaching, their content knowledge, and knowledge of institutional constraints. There were only two teachers involved in this study and they were both working in a similar environment, yet the differences in the ways they incorporated CAS were substantial. This foreshadows a wide variety of reactions as CAS becomes available across whole school systems. Differences in teachers' knowledge of mathematics and of institutional constraints and in their beliefs about learning and their purpose for teaching may be magnified in the presence of technology.

1.1 Teaching practices impact on student learning

It is important to study teaching practices because they impact on student learning. Therefore, the relationship between teaching practices and student achievement with technology is an important topic of current interest. For example, Keller, Russell, and Thompson (1999a, 1999b) attribute student success (i.e., improved conceptual understanding), on calculus courses taught with technology, partly to the adoption of student-centred teaching practices within a constructivist perspective.

Teaching practices are often attributed to teacher beliefs. For example, Jost (1992) explored teachers' beliefs about teaching (based on in-depth interviews) and compared them with their perception of the role of programmable graphics calculators. Jost categorized the teachers' perceptions of the role of the calculator along a continuum from a computational tool (for doing procedures) to that of an instructional tool (for promoting understanding). Jost found that the teachers who valued the calculator as a computational tool stressed content-orientated goals and viewed learning as listening. On the other hand she found that the teachers who came to appreciate the strength of the calculator as an instructional tool (to promote understanding) had student-centred views on learning (i.e., students can learn through interactions) together with interactive, inquiry-driven teaching styles. This phenomenon is also reported by others (Fine & Fleener, 1994; Simmt, 1997) and is consistent with the situation that is revealed in the present study.

1.2 Teacher knowledge influences teaching practices

Many attempts have been made to classify teaching practices and to relate them to teachers' knowledge and beliefs. Shulman (1986) distinguishes between teachers' content knowledge (understanding and organization of knowledge of specific topics) and pedagogical content knowledge (knowing ways of presenting knowledge to students, including representations, and understanding of what makes the learning easy or hard). Fennema and Franke (1992) describe other types of teacher knowledge: pedagogical knowledge (knowledge and planning of teaching procedures, behaviour management, and motivational techniques), knowledge of students, and knowledge of institutional constraints.

These types of teacher knowledge are interconnected (Even & Tirosh, 1995), making clear separations between them difficult. The teacher knowledge aspect of the interviews in the present study focused on content and pedagogical content knowledge but other knowledge (e.g., of institutional constraints) became evident.

Teacher knowledge in all senses is important because it impacts on teaching practice. For example, Jablonka and Keitel (2000) believe that teachers with highly developed content knowledge are ″more flexible in structuring content for teaching and in discussing students' ideas″(p.120). Gutstein and Mack (1998), in a study of teaching fractions, show that a teacher's teaching decisions and practices were derived from the depth of her interrelated knowledge of content, pedagogical content, and of students.

1.3 Identifying teaching practices

Many attempts have been made to classify different types of teaching practice. Kendal and Stacey's (notion of privileging (Kendal & Stacey, 1999, 2001) was useful for describing the teachers' behaviour that was observed during the teaching trials reported in this paper and was compatible with Kuhn and Ball's (1986) model for teaching practices.

1.3.1 Teacher privileging

In Kendal and Stacey (1999, 2001), we presented a way to describe the features relevant to the introduction of CAS in a teacher's individual way of teaching, using the concept of *privileging* (the word was first used by Wertsch, 1990). Privileging includes decisions about what is taught and how it is taught including: what is emphasized in the content (what is stressed and what is not stressed); what representations are preferred and ignored; the attention paid to procedures and concepts, rules, and meaning; and how much is explained or left to the students to work out for themselves. Privileging reflects the teacher's underlying beliefs about the nature of mathematics and how it should be taught. It is derived from an interplay of the teacher's beliefs and interrelated knowledge of the types in 1.2 above. It is moderated by institutional knowledge about students and school constraints and is manifested through teachers' practice and attitudes. Kendal and Stacey (1999, 2001) identified three *privileging characteristics* for each teacher that were relevant when they taught introductory differential calculus using CAS and that appeared to be influential in student learning. The teachers made choices about:

(1) Teaching approach (evidenced by teaching method and teaching style).

(2) Calculus content (evidenced by representations of differentiation taught and whether work was expected to be performed by-hand or by-CAS).

(3) Purpose of technology use (evidenced by the nature of the use of the CAS calculator).

1.3.2 Kuhn and Ball's model for teaching practice

In a comprehensive survey of teachers' beliefs and conceptions Thompson (1992) commended Kuhn and Ball's (1986) model for different teaching practices as ″constituting a consensual knowledge base regarding models of teaching″(p.136). Their model was useful in this study since it describes four teaching practices that were associated with particular beliefs about mathematics and goals for teaching. These teaching practices are:

″1. *Learner-focused*: mathematics teaching that focuses on the learner's personal construction of mathematical knowledge;

2. *Content-focused with an emphasis on conceptual understanding:* mathematics is driven by the content itself but emphasizes conceptual understanding;

3. *Content-focused with an emphasis on performance:* mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures; and

4. *Classroom-focused:* mathematics teaching based on knowledge about effective classrooms.″(p.2).

In this paper, teaching approach (see 1.3.1) is considered to have two usually complementary aspects, teaching method and teaching style that are associated with Kuhn and Ball's (1986) model. For example, 2 above involves the ″dual influence of the content and the learner. On the one hand, content is focal, but on the other, understanding is viewed as constructed by the individual″ (p.15). Thus, the teaching method is to promote understanding using a student-centred teaching style (where the teacher empowers the students to develop meaning for themselves). In contrast, for 3 above, the teaching method of the teacher is to promote knowledge of rules and procedures, presenting it in an expository manner such as lecturing, demonstrating explanations (i.e., a teacher-centred teaching style).

This paper reports the privileging characteristics of two teachers during their first and second experiences of teaching introductory differential calculus with a CAS calculator. During the second trial, the teachers made changes to their privileging and the teachers' reasons for these changes are also reported. How the teachers' privileging was related to their beliefs and knowledge is described; as are the factors that precipitated changes to the way the technology was used during the second trial.

2. Methodology

In two successive years, volunteer teachers, Andre and Benoit used CAS (TI-92) to teach approximately 22 lessons on introductory differential calculus to their Year 11 classes (16-17 year olds). They were both experienced teachers of mathematics, whose students had used graphics calculators in the classroom for several years. Andre (identified as Teacher A in some of the reported studies) and Benoit (or Teacher B) participated in the development of a teaching program (see Kendal & Stacey, 1999; Kendal, Stacey, & Pierce, in press a & b) that focused on a multiple representations approach to derivative. This developed the concept of derivative from three points of view: as a difference quotient and rate of change, as the gradient of a tangent to a graph, and from symbolic rules (involving the numerical, graphical, and symbolic representations respectively). Translations among the three representations were emphasized. Such an approach has been specifically recommended by Dick (1996) and is consistent with other approaches involving multiple representations (e.g., Tall, 1996; Repo, 1994).

Half of the lessons in the first year, and every lesson in the second year were observed and audiotaped. The methodology is fully reported by Kendal (2001). Teacher behavior was closely monitored (e.g., time spent on each type of differentiation activity, the nature of student-teacher interactions, and attitudes displayed towards calculus, technology, and the students). A comprehensive checklist of 52 observations was completed immediately after each lesson. Informal discussions with the teachers after lessons were also recorded. Finally, a privileging profile for each teacher was developed to describe each teacher's responses to teaching with CAS and to using a multiple representations perspective. It consisted of the three privileging characteristics and summarized the emphasis that they gave in their classes to: teaching approach (teaching method and style), calculus content (the representations of differentiation used and choice between by-hand/by-CAS on these representations), and the ways the CAS calculator was incorporated into their lessons (frequency and purpose for use). The purpose for calculator use was classified as functional (primarily to get answers), pedagogical (primarily for learning) following Etlinger (1974), or neutral (for checking by-hand solutions).

Nine months after the first trial and ten weeks prior to the commencement of the second trial, each teacher was interviewed separately to identify his personal knowledge of differentiation. This also provided a basis for comparison with the privileging that occurred during the second trial. During the interview each teacher was asked to demonstrate and discuss solutions to a set of calculus problems and to predict their students' responses. The teachers knew most of these problems would be included on the students' tests six months later. A wide spectrum of teacher characteristics was monitored. Broadly based on Shulman's (1986) and Fennema's (1992) classifications these included: content knowledge (personal knowledge of differentiation, preference for representation/s, alternative ways to solve differentiation problems); pedagogical content knowledge (awareness of alternative ways to teach differentiation problems); knowledge of technology (personal use of the CAS calculator); pedagogical knowledge (comments regarding teaching methods and teaching styles, confidence in teaching with the CAS calculator), knowledge of students; and institutional knowledge (awareness of subtle school pressures and explicit constraints).

Ten weeks after the second teaching trial, a second teacher interview was conducted to corroborate the privileging identified by observation of lessons during the second trial. Teachers were asked to reflect on their teaching practices, particularly on how they had used the CAS calculator to support conceptual understanding of the concept of derivative.

3 Results: Factors that influenced teaching behaviors

The following section summarizes the observations about the teachers' behavior from all of the sources described above.

3.1 Teachers' knowledge

3.1.1 Teacher's own knowledge of differentiation. Overall, Andre displayed less depth and less integrated knowledge about the concept of derivative than Benoit, who displayed deep and integrated knowledge. During the first interview Andre demonstrated how to differentiate functions symbolically (using algebraic rules) and graphically (by finding the gradient of the tangent drawn on a curve), and he could translate between these representations, interpreting either one in terms of the other. He did not use the numerical approach (involving difference quotients of function values). Benoit understood all of this and readily translated between all three representations. During the interview, he regularly demonstrated alternative solutions to problems drawing on the different representations. Andre showed less variation in solutions and had difficulty solving some of the problems.

3.1.2 Teachers' institutional knowledge. During both teaching trials, the teachers knew that their students could use the CAS calculators only for the tests associated with the trials. They would not be available for the formal school examinations to be held in three months time or on the official state school examinations in fifteen months time. They were also aware that the style of assessment used for the first trial was similar to the official school examination (based mostly on the symbolic derivative) whereas for the second trial, the assessment involved formulating and interpreting derivatives from tangents, tables of values, and symbolic expressions. This was new institutional knowledge for the second trial.

3.2 Teachers' privileging characteristics

This section describes three privileging characteristics of the teachers, so that the different emphases that they gave to the classroom can be understood. Two of them did not change: teaching approach (3.2.1) and purpose of technology use (3.2.1), whereas the third characteristic did change: calculus content (3.2.2).

3.2.1 Privileging related to teaching approach

Teaching method: Andre's focus was on teaching rules and strategies for carrying out procedures and during both interviews he talked primarily about routines to solve problems. In contrast, Benoit's teaching emphasis was on understanding the concept of derivative. He gave the meaning to the symbolic derivative as gradient of a curve, often depicting the gradient of the tangent to the curve at a point using his outstretched arms to represent the tangent line (an enactive representation). He also encouraged the students to use visualization techniques to interpret mathematics, such as visualizing a graph where the tangent has slope zero. During the first interview he solved each problem several ways, explained his use of different representations, and by making connections amongst ideas and results, convinced himself that his answer was appropriate. During the second interview he talked about conceptual understanding:

"Getting the tangent idea through to them, what the gradient actually represents, what the derivative actually represents, and the relationship between them - I think we've done that very nicely with the calculator.″

Teaching style: Andre adopted a teacher-centred style. He mostly lectured his students who were expected to copy down his lesson notes and lists of CAS key strokes. In contrast, Benoit adopted a student-centred teaching style. He guided class discussions between 'each student and teacher' and 'student and student' and he encouraged the students to construct meaning for themselves.

Thus, Andre's teaching approach, which emphasized student performance and mastery of mathematical rules through teacher-centred lectures, is identified as ″content-focused with an emphasis on performance″ (using Kuhn and Ball's 1986 model). Benoit's teaching approach, which emphasized conceptual understanding of content and student construction of meaning through student-centred class discussion, is identified as ″content-focused with an emphasis on conceptual understanding″.

3.2.2 *Privileging of calculus content*

During the first teaching trial and in the first interview, Andre focused almost exclusively on symbolic differentiation. However, during the second trial he expanded his teaching to include the calculation of derivatives at a point from graphical and the numerical representations of the function. This came about after the first interview when he realized that the students' second trial assessment would involve all three representations, unlike the first trial assessment that was essentially symbolic.

During both trials, Benoit consistently stressed the symbolic derivative and he used a graphical representation to give the symbolic representation meaning as discussed above. Although he personally demonstrated the ability to obtain an instantaneous rate of change at a point numerically (finding a rate of change or a difference quotient) during the first interview, he actively rejected teaching about difference quotients in the second trial. He explained this was because his students were a low-attaining group and would not cope with the three representations of derivative (i.e., he made changes to the calculus content he taught in response to knowledge of his second cohort of students).

The second aspect of privileging for content choice is whether students were encouraged to solve problems using CAS or by-hand. In both trials, Andre appeared comfortable for students to make their own choices about using or not using CAS; methods using CAS were demonstrated and students were free to use them. In contrast, Benoit encouraged CAS for graphing only and actively discouraged it for symbolic procedures, especially in the second trial.

3.2.3 Privileging of purpose of technology use

Technology use is characterized by the purpose (nature) of its use. In the first teaching trial, Andre regularly linked the CAS calculator to an overhead projector and frequently demonstrated symbolic procedures to the students and allowed them to use CAS freely. He avoided using graphs and tabular representations. In the second trial, he again used the overhead projector of the CAS screen in most lessons. He taught his students the additional CAS numerical and graphical differentiation routines, to obtain derivatives at a point. This is described, following Etlinger (1974), as *functional* use.

Andre showed little pedagogical CAS use in either trial. In the first interview, he reported one instance:

″I'd say, when you see these words [average rate of change] it means between two points, and when you see this word [instantaneous] that means at a point . . . [I am] giving them strategies . . . and we did it [used a dynamic graphing program] to understand the straight line against the curve.″

In the first teaching trial, Benoit used the technology freely to draw graphs but he noticeably controlled student use of the CAS calculator for symbolic algebra procedures. In the second trial, he actually reduced his functional use of the CAS calculator (and he discouraged his students). Only pedagogical use of the symbolic capability was encouraged. This occurred when he believed CAS use would promote understanding by providing data for exploratory activities. An example was when students used the CAS to build up a table of derivatives of polynomials from which the general rule for differentiating a polynomial could be induced. Some lessons are described in detail in Kendal & Stacey (2001).

Benoit maintained his emphasis on linking the symbolic and graphical representations to help the students attach meaning to the symbolic derivative.

″It's [the CAS] good for discovery because it takes a lot of the hack work out of the teaching for understanding but you still need to teach pen and paper skills. I think there are certain skills that the kids have to have, even if you can use the technology to do it. I think the kids have to have the [symbolic manipulation] skills as well, without the technology. I think that's essential for their understanding. It's not sufficient to just use the calculator, they have to have the understanding of what's behind it.″

Thus, during both trials, Andre used the CAS calculator primarily for functional purposes eventually involving the three representations. Benoit also used it functionally, (graphically but not symbolically), and pedagogically (graphically for illustrating the meaning of derivative and symbolically for pattern finding purposes). Although neither teacher changed their purpose for using the CAS calculator, they both used it in new ways in the second trial to accommodate the changed emphasis they gave to the representations (i.e., calculus content) as discussed in 3.2.2.

3.3 Changes in teachers' privileging characteristics

Table 1 summarizes the teachers' privileging characteristics during observations of the first trial's lessons (reported by Kendal & Stacey, 1999) and substantiated by the first teacher interviews (see Kendal 2001). Table 2 reveals the changes in privileging that occurred during the second trial. The tables show that the teachers did not change their privileging of teaching approach or purpose of technology use; these privileging characteristics were stable over two years. However, the third characteristic, calculus content, was susceptible to change. In addition to using the symbolic (S) representation Andre introduced the graphical (G) and numerical (N) representations whereas Benoit reduced his use of N while continuing to use the S and G representations. He also reduced his already low approval of by-CAS methods for symbolic questions.

Table 1 *Andre's and Benoit's Privileging in Trial 1*

Privileging Characteristics (First Trial)	
Andre	Benoit
Calculus content	
Preferred S	Preferred S Used G Linked S & G
CAS approved for S,G,N	CAS approved for G
Teaching approach (method and style)	
Rules for routine procedures	Promoted understanding
Teacher-centred lectures	Student-centred discussion
Purpose of technology use	
Functional (often used)	Pedagogical (rarely used, except for G)

Table 2 *Changes in Andre's and Benoit's Privileging in Trial 2*

Changes in Privileging Characteristics (Second Trial)	
Privileging Characteristics (First Trial)	
Andre	Benoit
Calculus content	
Introduced G Introduced N Linked N & G	Reduced use of N
	CAS not allowed for S
Teaching approach (method and style)	
No change	No change
Purpose of technology use	
No change	No change

4 Teacher privileging was underpinned by beliefs and knowledge

4.1 Beliefs about learning

Andre's belief about learning was that students needed to learn to recognize relevant rules and know how to carry out the related procedures. He stressed these points in his interviews. During both trials he emphasized rules and demonstrated (with a teacher-centred teaching style) both by-hand procedures and CAS procedures. In the first trial, Andre, with less comprehensive content knowledge, privileged symbolic differentiation because he knew that the procedures were exact (stated in the first interview). During the second trial he found out that exact values of the derivative at a point could be found simply on the calculator by graphing a tangent to a function and asking for its gradient or by calculating a difference quotient using two very close points. In fact, these are approximate rather than exact values, but Andre did not fully understand this difference. In consequence, he encouraged his students to carry out differentiation procedures (efficiently and accurately) in the symbolic, numerical (from a difference quotient) and graphical (from a tangent) representations using CAS. This was not primarily done to demonstrate the links among gradients, difference quotients, and symbolic derivatives, but mainly in order to solve standard problems using derivatives at a point. Thus, across the two trials, Andre's teaching approach was stable and reflected his beliefs about learning revealed in the interviews.

During both trials Benoit believed it was his responsibility to foster student understanding and his belief about learning was that students needed to understand the underlying mathematical concepts, not merely to know how to carry out the procedures. His deep content knowledge empowered him to guide meaningful student-centred class discussions during which he assisted each student to develop understanding. He privileged the symbolic representation believing it to be the most powerful and useful but was willing to use other representations to enhance students' understanding as discussed above. Benoit generally discouraged the students from using the symbolic algebra capability of CAS, only encouraging them to use it for whole-class pedagogical purposes e.g. generating results that would subsequently be used for guessing differentiation rules. Benoit used CAS principally to promote students' understanding, which is consistent with his beliefs about learning. Thus, across the two trials, Benoit's teaching approach and purpose for using CAS were stable and reflected his beliefs about learning.

In summary, each teacher's privileging of teaching approach (method and style of teaching) and purpose for use of CAS were consistent during the first and second trials and reflected each teacher's individual view about learning.

4.2 Purpose for teaching

During both teacher interviews Andre revealed that he believed that his main purpose for teaching was his responsibility to help his students pass examinations. During the first interview, which occurred between the two teaching trials, Andre realized for the first time that the second trial assessment would involve multiple representations of differentiation. He responded to this ″new″ institutional knowledge by expanding his repertoire of calculator procedures. For example, he showed the students new CAS procedures such as substitution of x values into functions to find ordered pairs that could be used to create a difference quotient calculation (an approximation to a numerical rate of change and an ″excellent″ approximation to the gradient of the tangent/curve). Andre led his students to believe they were both exact derivatives (see discussion in 4.1). Thus in the second trial, in addition to symbolic representation he privileged the numerical and graphical representations of differentiation. Andre changed his privileging of calculus content in response to his changed institutional knowledge, i.e., a new perception of what the students needed to know to be successful on the second trial examination. However, he made the changes in a way that was consistent with his beliefs about learning.

Similarly, Benoit revealed that his main purpose for teaching was his responsibility to help his students pass examinations. His second trial class had a majority of academically weaker students, so he reduced the content they were taught. He avoided teaching about differentiation using a numerical (tables) representation, believing that the students would not cope with three representations. He also strongly controlled their opportunity to individually use symbolic algebra believing that they needed practice using by-hand symbolic differentiation procedures both for understanding and to be able to cope with future examinations. Thus, Benoit changed his privileging of calculus content in response to his changed perception of what the students needed to know to be successful on the second trial examination. However, he rationalized the changes in a way that was consistent with his beliefs about learning, where teaching for understanding was dominant.

Thus, during the second trial both teachers maintained their views about their purpose for teaching but they changed their perceptions of the mathematical content the students needed to know to be successful on examinations. Both teachers used CAS to support the representations the teachers now considered important but in ways that were consistent with their beliefs about learning described above. Both teachers, in response to new knowledge, made changes to how they used CAS in ways that were consistent with their own beliefs and knowledge. These results are consistent with other research. For example, Tharp, Fitzsimmons, and Brown Ayers (1997) showed that the teaching style tended to be unchanged by the addition of technology. In their study, after an initial attempt at a more enquiry based learning using a graphical calculator, teachers with a rule-based (procedural) view tended to revert to their procedural view style of teaching, while teachers who were not rule-based remained more likely to focus on student conceptual understanding and thinking.

4.3 Institutional constraints

As just described, both teachers were influenced by their belief that the main purpose for their teaching was to assist students pass examinations. The knowledge that their students had to sit future examinations without CAS was one factor that prevented them from using fully the symbolic capabilities of the CAS calculator. The teachers responded differently to this constraint. Andre was more accepting of the aims of the research project, allowing the students to use CAS and adding new techniques in response to new knowledge of expectations of students. Benoit was always more concerned that the students developed by hand skills and rejected some of the aims of the research trial for a class he assessed as mathematically weak.

In summary, Figure 1 displays the four factors that appeared to determine what Andre and Benoit taught in this study and how they taught it:

1. Beliefs about learning, manifest through the privileging characteristics of teaching approach and purpose for using technology (shown shaded) that were stable.
2. Beliefs about purpose for teaching, manifest through the privileging characteristic of content choice (shown shaded) that was not stable.
3. Teacher’s content knowledge.
4. Knowledge of institutional constraints.

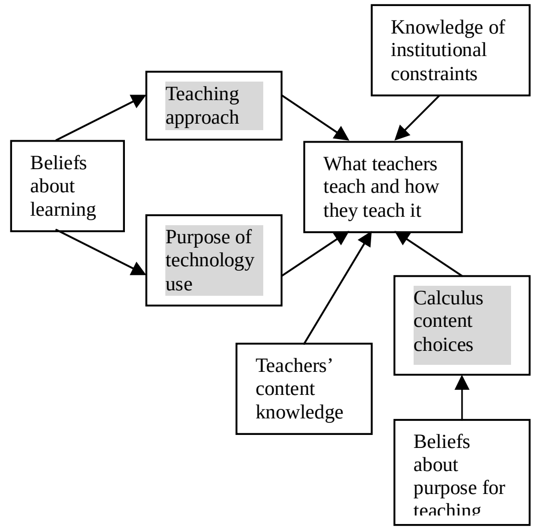

Figure 1. Factors determining what Andre and Benoit taught and how they taught it.

5 Implications

During both trials the teachers taught in ways that reflected their personal content knowledge, their beliefs about learning and purpose for teaching, and their individual reactions to institutional constraints. In turn, these characteristics impinged on the ways the teachers incorporated CAS into their lessons. It is commonly believed that teaching practices impact on student learning outcomes and our other evidence from this study indicates that this was the case in trial 1 (see Kendal & Stacey, 1999) and in trial 2 (see Kendal 2001) where the students in different classes were successful on different categories of test items that appear to be dependent on their teachers’ privileging. This is in a context of similar overall levels of achievement.

The teachers’ beliefs about learning and their purpose for teaching seem to have underpinned their privileging characteristics and were influential in the changes that occurred during the second trial. This raises many issues. First, what privileging characteristics are most beneficial to student learning in a CAS supported classroom? If students achieve better understanding when their teacher employs student-centred-teaching styles using CAS to promote understanding (or indeed if another teaching approach is found to be superior), how will the teachers with contradictory underlying beliefs be able to change to accommodate new teaching practices that are incompatible with their beliefs? Alternatively, different teachers may be differentially successful with teaching approaches, so that maximizing effectiveness of teaching with CAS may be best achieved by advocating different styles for different teachers. This study has shown that the teachers were able to change the mathematical content they taught when the changes helped them better achieve their purpose for teaching. However it is anticipated that teachers will find it more difficult to change their teaching approach and purpose for use of CAS (privileging characteristics) without changing their beliefs about learning and, in some cases, expanding their content knowledge.

Secondly, how does CAS become legitimized within the school culture? This is not a simple question, because it is not agreed what nature of use should be legitimized. Our two teachers drew their positions at least in part from the institutional position, which did not legitimize CAS. It will be very interesting to see if teachers make the changes more easily when CAS use has become institutionalized as in our new study, which examines the first cohorts of students in an examinations system where CAS is allowed (Stacey, McCrae, & Asp, 2000). However, there are deeper debates which arise when the institutional constraints do legitimize CAS, concerning what really constitutes doing mathematics; the right balance between by-hand and by-CAS algebraic skills to achieve various goals and the appropriateness of using CAS to compensate for inadequate algebraic skills. In addition, there is a lack of legitimacy that arises not from institutional constraint but from a need for new pedagogical content knowledge. In our study, the existing curriculum was enhanced by CAS, principally through its ability to improve understanding by enabling a multiple representations approach. However, because the existing curriculum goals were unchanged there was no real need for the symbolic manipulation power of CAS – the mathematics stayed within the expected range of by-hand skills. New pedagogical knowledge was required to know how to make good use of the new power.

A final observation from the study is to comment on the recommendation that calculus should be taught from a multiple representations perspective. We regard the rejection by Benoit of this in the second trial as strong evidence that learning can be confused if too many representations are introduced and that a streamlined approach, selecting between methods, is a practical necessity. This supports a recommendation of Guin and Trouche (1999), that the explosion of methods available with CAS needs to be institutionally controlled.

References

Dick, T. P. (1996). Much more than a toy. Graphing calculators in secondary school calculus. In P. Gomez & B. Waits (Eds.), *Roles of Calculators in the Classroom (pp.31-46).* [On line]. Available: http://ued.uniandes.edu.co/servidor/em/recinf/tg18/Dick/Dick-1.html (Downloaded 18th October 1998).

Etlinger, L. (1974). The electronic calculator: A new trend in school mathematics. *Educational Technology, XIV*(12), 43-45.

Even, R., & Tirosh, D. (1995). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject matter. *Educational Studies in Mathematics, 29,* 1-20.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (pp.147-164). New York: Macmillan.

Fine, A. E., & Fleener, M. J. (1994). Calculators as instructional tools: Perceptions of three preservice teachers. *Journal of Computers in Mathematics and Science Teaching, 13*(1), 83-100.

Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning,* 3 195-227.

Gutstein, E., & Mack, N. K. (1998). Learning about teaching for understanding through the study of tutoring. *Journal of Mathematical Behavior*, 17(4), 441-465.

Jablonka, E., & Keitel, G. (2000). The distinctive professional knowledge and professionalism of teachers of mathematics: the problem of integration of knowledge. In J. Wakefield (Ed.) *Mathematics shaping the future* (pp. 118-124). Melbourne: MAV.

Jost, K. L. E. (1992). *The implementation of technology in the calculus classroom: An examination of teacher beliefs, practice and curriculum change.* Unpublished Ph.D., (Syracuse University, 1992). Dissertation Abstracts International, 49/06 1397.

Keller, B., Russell, C., & Thompson, H. (1999a). A large scale study clarifying the roles of the TI-92 and instructional format on student success in calculus. *The International Journal of Computer Algebra in Mathematics Education, 6*(3), 191-207.

Keller, B., Russell, C., & Thompson, H. (1999b). Effects of student-centred teaching on student evaluations in calculus. *Educational Research Quarterly, 23*(1), 59-73.

Kendal, M. (2001). *Teaching and learning introductory differential calculus with a computer algebra system*. Unpublished Ph.D. Thesis. (The University of Melbourne).

Kendal, M., & Stacey, K. (1999). Varieties of teacher privileging for teaching calculus with computer algebra systems. The *International Journal of Computer Algebra in Mathematics Education, 6*(4), 233-247.

Kendal, M., & Stacey, K. (2001). The impact of teacher privileging on learning differentiation with technology. *International Journal of Computers for Mathematical Learning. 6*(2), 143-165.

Kendal, M., Stacey, K., & Pierce, R. (in press a). L'influence des environnments de calcul formel sur les modes de travail des enseignants. In D. Guin & L. Trouche (Eds.) *Calculatrices symboliques. Transformer un outil en un instrument du travail mathematique: Un probleme didactique* Grenoble, France:La Pensee Sauvage.

Kendal, M., Stacey, K., & Pierce, R. (in press b). Teaching with CAS in a time of transition. *International Journal of Computer Algebra in Mathematics Education*.

Kuhn, T., & Ball, D. (1986). *Approaches to teaching mathematics: Mapping the domains of teacher knowledge, skills and dispositions.* East Lansing: Michigan State University, Center on Teacher Education.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meaning: learning cultures and computers*. London: Kluwer Academic Publishers.

Repo, S. (1994). Understanding and reflective abstraction: Learning the concept of derivative in a computer environment. *International DERIVE Journal, 1*(1), 97-113.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher* (February), 4-14.

Simmt, E. (1997). Graphing calculators in high schools. *Journal of Computers in Mathematics and Science Teaching, 16*(2/3), 269-289.

Stacey, K., McCrae, B., & Asp, G. (2000). Goals for a CAS-active senior mathematics curriculum. In M. Thomas (Ed.), *TIME 2000* (Proceedings of the International Conference on Technology in Mathematics Education, 11-14th December, pp.244-252). Auckland, New Zealand: The University of Auckland.

Tall, D. (1996). Functions and calculus. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp.289-325). Dordrecht: Kluwer Academic Publishers.

Tharp, M. L., Fitzsimmons, J. A., & Brown Ayres, R. L. (1997). Negotiating a technological shift: Teacher perception of the implementation of graphing calculators. *Journal of Computers in Mathematics and Science Teaching, 16*(4), 551-575.

Thompson, I. G., (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.127-146). New York, NY: Macmillan.

Wertsch, J. (1990). *The voice of rationality in a sociocultural approach to mind*. Cambridge: Cambridge University Press.

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