Trends in Researching and Teaching Problem Solving in School Mathematics in Australia: 1997 – 2000.

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Overview

This paper presents a brief summary of recent trends in the role of problem solving in school mathematics in Australia. Australia has a federal system of government and there are 8 independent major education systems. Policies about mathematics education and their implementation vary among these systems, although there are many commonalties. Generalisation is difficult and the observations following are not uniformly applicable to all systems. However, there is substantial sharing of ideas and some national agreed policies, which makes a report on Australia feasible.

Making students better problem solvers was formally established as a goal of the mathematics curriculum in every state and territory in Australia during the 1980s (Stacey & Groves, 1990). In practice, this goal encompassed two somewhat distinct aspects. Firstly, there is the goal of improving the capacity of students to solve problems, in particular problems in a real world context. This goal has been addressed by ensuring that students have some genuine experience of substantial problem solving by being given the opportunity to solve real world problems at school, and also by giving some instructional attention to the processes of problem solving including helpful strategies for approaching problems. This aspect of the problem solving goal has received less attention in recent years. The second aspect of the problem solving goal is to teach mathematics through problem solving approaches. Nisbet and Putt (2000) observe that this aspect of the problem solving goal has been given impetus in recent years by the popularity of constructivist theories to guide teachers as facilitators of learning.

In recent years, government initiatives in mathematics education in most education systems of Australia have emphasized the importance of establishing basic educational foundations for all students, especially in literacy but also in numeracy. The emphasis on monitoring achievement on basic skills might have excluded any emphasis on problem solving. This, however, has not happened. Although not receiving the prominence that it had several years ago, problem solving has continued to be regarded at least as an essential competency in mathematics. Nisbet and Putt (2000) attribute this to two causes. Firstly, there was the identification of "using mathematical ideas and techniques" as one of a small number of "key competencies" for national vocational education (Mayer, 1992). These key competencies were intended to describe the broad outcomes of education for effective citizenship and for work, and so the emphasised applying knowledge across the curriculum and being able to solve problems. The second factor identified by Nisbet and Putt was the prevalence of constructivist philosophies, interpreted broadly, within the research community and also in professional development for teachers. Since students must ultimately construct their own mathematical knowledge, a crucial role for the teacher is to offer the problems and learning experiences which provide students with the raw material for constructing mathematical concepts.

Within the research community, problem solving has received further impetus from the study of small group learning processes. On-going research into the use of collaborative learning (of various types) in mathematics classrooms has continued to highlight the way in which students solve problems, how they come to construct knowledge through this process and the need for metacognitive awareness of problem solving processes.

The introduction of new technology, (especially graphics calculators) has been a major concern in recent years and there has been a great deal of exploration about the possibilities that this offers for new types of problems to be tackled, especially in senior secondary school mathematics (Stacey, 2000). Graphics calculators, spreadsheets and data logging equipment are widely used in some but not all states. They have opened new possibilities for gathering real data (e.g. about speed, light intensity or temperature), fitting a function to create a mathematical model and using the model to make predictions or explain phenomena. Widespread availability of statistics software (handheld and on school computers) has also stimulated interest in solving problems involving real data. Students conduct surveys, or gather data from published sources and increasingly the internet, to bring topical questions into the classroom. Although students are still taught to compute statistics by hand, the use of statistics software to analyse substantial data sets has further opened the classroom to the real world.

The remainder of this paper discusses two features of problem solving in Australia: the use of substantial real world modeling and problem solving investigations into high stakes assessment and the insight into problem solving from the study of small group processes. There is also substantial research being undertaken in Australia into the way that students solve problems in different content areas of mathematics, some of which is reviewed by Nisbet and Putt (2000), but is outside the scope of this paper. Nisbet and Putt (2000) and Nisbet, Putt and Taplin (1996) both provide extensive reviews of Australian research on problem solving for four year intervals. Keeves and Stacey (1999) report other aspects of problem solving, in the context of broad developments in research in mathematics education in Australia since 1965.

Real world modeling and problem solving in Victoria

In Victoria (one of the Australian states), there was a significant attempt to promote the teaching of mathematical modeling and investigation in schools by introducing substantial problem solving into the formal assessment of all mathematics subjects from 1989. This assessment is extremely politically sensitive as decisions about access to university places are made on the results. There were several arrangements for different subjects. In one subject, there was a project (generally real world mathematical modeling) on a “theme” set by the external examiners. This was expected to take 20 hours (in practice students spent much longer) and a written report of 2000 words was assessed by teachers with official guidance and monitoring. In another, there was a substantial problem solving task (somewhat more closed than the project), designed to take about 8 hours, again assessed by a written report and again allowing considerable possibilities for extensions according to the students’ own choice. These arrangements had an immediate and substantial impact on the teaching of mathematics in junior secondary years. However, there was constant pressure for change arising from (a) allegations that sometimes take-home work was being done by people other than the students themselves, (b) the difficulty of confining students' efforts (including word processing etc) to the allotted time (c) lack of confidence that grades given by different teachers and schools on an open task were really comparable. Throughout the decade, modifications were regularly made to meet these pressures and these are described by McCrae & Stacey (1997). In 1998, Stacey and McCrae described changes that markedly constrained the tasks, including the addition of short tests based on the take-home problem and project topic. They concluded that the concerns had been addressed, that students still found the problem solving task challenging and that it provided an experience of working intensely and creatively on a problem that was unique in their thirteen years of school mathematics education. From 2000, however, problem solving and project work are no longer required. Control has been passed to schools in an effort to streamline assessment and reduce what have been seen as excessive demands upon students and teachers. Many schools have chosen to replace it by short answer tests, which directly prepare students for the external examination of facts and skills. A bold experiment of assessment driven change and a real focus of problem solving and modeling in the curriculum thus comes to an end.

Those who believe that assessment can drive the curriculum should not under-estimate the potential of traditional practices in the school system to mould the assessment. There is now much less opportunity for students to engage in substantial problem solving in Victorian schools and, to a large extent, this must be seen as a move away from serious attention to the goal of making students better problem solvers. Instead, what energy is being put into the problem solving goal is being channelled into the new opportunities for viable tasks presented by new technology and, to some extent and in individual classrooms, into the second aspect of the problem solving goal - to teach through problem solving.

Insights from research on small group work.

Teaching through problem solving has been energized in the 1990s around Australia by the call to use small group work and investigative approaches for teaching mathematics. Recording and analyzing small group work in detail has provided a window for the study of how students think about mathematical problems and the way in which breakthroughs occur or obstacles block progress. This work has been carried out in classrooms from the earliest years of schooling to senior secondary mathematics. Considerable documentation has now occurred of the characteristics of tasks that stimulate productive classroom investigations. Stillman (1998), for example, examined how students' background knowledge of the context of a task affects their approaches to the task and their engagement with it. Williams (2000) has documented the beneficial effects of using tasks which can provoke "discovered complexity" and Geiger and Goos (1996) reported characteristics of problem tasks that affect the degree of collaboration between students. All of these studies are with senior students. Working with Grade 5/6 students, Walta has explored the potential of innovative cross-curricula Microworlds programming tasks to stimulate understanding of general problem solving strategies. She also established that the teacher needs to give explicit attention to transfer if benefits are to flow to specific subject areas such as mathematics. Thomas and Holton in New Zealand have looked at the structures in classrooms of very young children to establish the working conditions that promote good engagement with mathematical ideas.

One example of a research study, which aims to examine the conditions for classroom practice which promotes students learning and problem solving is provided by Williams (Williams, 2000). She studied the links between sustained engagement and conceptual development when her own senior secondary mathematics students worked collaboratively to solve an unfamiliar challenging problem. Her class collaborated at two levels within the classroom; each small group collaborated to solve the problem but also provided feedback to the class as a whole at regular intervals. To study engagement, Williams built upon Csikszentmihaly's concept of Flow (Csikszentmihalyi, 1992) by formulating the concept of a Discovered Complexity as a time when the conditions for Flow were present. Flow is an optimal learning condition that exists when a person or group are so involved in the task they lose all sense of time and self, working just above their present skill level on a challenge almost out of reach. (Csikszentmihalyi, 1992). Williams' Discovered Complexity, a complexity that is not evident to students at the start of the task but arises during task completion, exists when a group spontaneously focuses on a question formulated within the group and all group members use new skills and concepts to resolve this question. Williams demonstrated the frequent discovery of complexities led to enhanced conceptual development for students. The frequency with which collaborative groups discovered complexities was linked to the degree of common background shared by the students and the interaction patterns within the collaborative group. By definition, a Discovered Complexity opens overlapping Zones of Proximal Development for the students in the collaborative group (Vygotsky, 1978) because they work together on mathematics new to all group members. Williams hypothesises that the creation and sustenance of overlapping Zones of Proximal Development is dependent upon the group sharing a common background understanding of the mathematical scaffolding required to undertake the task and that the group interaction patterns must support a shared development of new understandings.

Goos (2000a; 2000b) also studied group interaction patterns in a senior secondary classroom designed to foster and sustain mathematical thinking. She conducted a three- year research study that investigated patterns of social interaction associated with metacognitive activity and analysed the teacher's role in creating a classroom culture of collaborative inquiry that supported students' mathematical thinking (see Goos, Galbraith & Renshaw, 1999). The theoretical framework for the study drew on sociocultural theories of learning, and extended Vygotsky's (1978) notion of the zone of proximal development (ZPD), originally conceived in terms of expert-novice interactions, to include collaborative partnerships between learners with incomplete but comparable expertise. Students were videotaped as they worked together on challenging problems set by the teacher as part of their regular mathematics lessons, and transcripts of their conversations were analysed to identify the metacognitive function and collaborative structure of the dialogue.

Students' metacognitive activity was found to be organised around both routine monitoring of progress and their recognition of, and response to, various types of obstacles. These difficulties were signalled by metacognitive "red flags", such as lack of progress, a calculation error, or an anomalous result. Analysis of successful problem solving episodes indicated that collaborative metacognitive activity was characterised by students' mutual engagement with each other's ideas, in the form of transactive reasoning (Kruger, 1993). Transactive statements, questions, and responses may be directed at clarifying, elaborating, justifying, or critiquing one's own or a partner's thinking. This was observed in the way that students:

- explained their own thinking for the benefit of a partner
- asked a partner for feedback on their own ideas, or for help in finding errors
- checked their understanding of a partner's thinking, for example by asking for explanations or evaluating their partner's strategies.

However, problem solving in small groups was not always successful, and analysis of transcripts of unsuccessful collaboration showed how poor metacognitive decisions led students to impose inappropriate conceptual structures on a problem, overlook calculation errors, or mistakenly reject a correct answer (e.g. Goos, 1998). Interestingly, there was a similar proportion of metacognitive dialogue in both successful and unsuccessful problem solving sessions. However, unsuccessful collaboration was marked by a lower incidence of transactive discussion around, and generated by, metacognitive ideas. In practice, this was observed when students passively accepted unhelpful or misleading ideas and ignored potentially useful strategies suggested by peers. By contrast, successful problem solving was favoured if students challenged and subsequently discarded unhelpful ideas and actively endorsed fruitful strategies. Being held accountable by peers for explaining "how" and "why" may have prompted students to explore an ideas more thoroughly, or to step back from a task and recognise a mistake. This links to an early study of Stacey (1992) which demonstrated that groups of problem solvers often did not capitalize on good ideas offered in the group because of lack of engagement with each others ideas. Successful problem solving groups checked the ideas of others and did not quickly seek simple ideas which everyone could "understand" in a superficial way.

Both the Wiliams and the Goos studies show how teachers may support learning through collaborative problem solving by encouraging students to explain and justify their thinking, ask a partner for feedback or help, comment thoughtfully on a partner's work, and ask for explanations if they do not agree with or understand their partner's thinking. Both studies highlight the critical role of the teacher in establishing a classroom environment conducive to this type of student interaction They both provide some examples of how real progress is being made into creating classroom environments that can effectively use an investigative approach to learning mathematics.

Conclusion

There has been a change in the main interpretation of the goal of problem solving in school mathematics in Australia. Ambitious attempts to assess problem solving and to encourage the teaching of problem solving (through both experience and reflection) have been abandoned or reduced, largely in response to external pressures. On the other hand, there are encouraging signs that the use of a problem solving approach to teaching is becoming much more sophisticated. In-depth research is providing guidance on how to replace traditional teaching with approaches that engage students more fully in mathematical thinking.

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