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Photo file: kayestacey2.jpg

NOTE; We could have hyperlinks to the underlined words if you wish.

Kaye Stacey is Foundation Professor of Mathematics at the University of Melbourne and head of the Department of Science and Mathematics Education. She is well known nationally and internationally as an author, researcher, teacher and teacher educator. Kaye believes that new technology is the most significant driver of change in mathematics teaching for this generation, providing an imperative to adjust curriculum as well as new opportunities for teaching mathematics better. University of Melbourne research into technology in mathematics began with exploration of the potential of spreadsheets and function graphers; work which later lead to the phased introduction of graphics calculators into senior mathematics exams, supported by substantial professional development for teachers. The team is now looking ahead to the possible use in senior mathematics of software and calculators capable of computer algebra. This will require a substantial rethink of senior mathematics. Kaye Stacey has an honours degree from UNSW and a doctorate in pure mathematics from Oxford University. Her books include *Thinking Mathematically* (Mason, Burton, Stacey; 1982; Addison Wesley) and *Graphic Algebra* (Asp, Dowsey, Stacey, Tynan; 1997; Curriculum Corporation).

ATTACH PHOTO

Telling the whole of the maths story:

how can we keep the plot with graphics calculators?

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In this presentation, I wish to make two points and to illustrate them with two examples. The first point is that new technology is providing us with wonderful new opportunities to improve the teaching and learning of mathematics. There are opportunities at every year level and through every facility on the new calculators: the home screen, graphing, statistics, programming and, on newer models, symbolic manipulation for algebra. My two examples illustrate simple activities that provide greatly enhanced learning opportunities. Activities like these are now commonplace amongst teachers using graphics calculators, but it is useful to consider what makes them significant
On the other hand, I see that the major issue for the profession as graphics calculators become widespread is to prevent Mathematics becoming too empirical. Leading mathematics teachers will have to guard against a tendency towards too much data collection (whether from real situations or from number patterns) and insufficient analysis and reasoning. How empirical is too empirical is of course a matter of debate In the final section, I will invite you to respond to the dilemmas that face us.

Example 1. Learning algebra, not just guessing patterns.

The first example is for early secondary school, where initially teachers and parents may think there is little benefit from having a graphics calculator. However, the big home screen has great value.

Students can look for patterns when they follow simple procedures such as the following:
"Think of a number, add 2, multiply by 5 and subtract 4 times the number you first thought of".

Using a graphics calculator enables entry errors to show and be corrected and provides a record of recent results, ideal for making and testing conjectures. Most students undertaking a problem such as this will start by trying some numbers – an excellent problem solving strategy to begin to see what is happening. They may work like Lucy has done:

LUCY
Try 7, 7 + 2 = 9, 9 x 5 = 45, subtract 28 gives 17. I don't notice anything.
Try another number, say 3.
This gives 3 + 2 = 5, 5 x 5 = 25, subtract 12 gives 13.
Now I think I see a pattern. You add ten.

To this point, the calculator has been of little advantage. Students certainly should be able to do arithmetic like this mentally. In fact, calculator use may be a disadvantage as some students might record the sequence of button pushes as

7+2 = 9 x 5 = 45 – 28 = 17

The roots for recording in this way come from students' early experience of the equals sign as meaning "work it out now and write the answer on the other side". From the earliest years at school, teachers write "sums" such as 8 – 3 = ….. and expect children to interpret the equals sign as an instruction to work out the answer and as a guide to where to write it. Teaching students to use the equals sign as an indicator of equality of both sides (and nothing else) is critical to learning algebra (Stacey and MacGregor, 1997a). Observing that the calculator button that means "work it out now" is labelled as EXE or ENTER etc, rather than as "=" is a helpful discussion starter.

The greater value of the calculator comes after this stage of data gathering and pattern spotting. Students in early secondary school must widen their idea of what a number is: they need to expand the range of numbers with which they feel familiar away from just the positive whole numbers. They need to explore their properties, so we can encourage Lucy to use her calculator to try different numbers:

7.89201 + 2 = 9.89201, 9.89201 x 5 = 49.46005, subtract 31.56804 gives 17.89201

The pattern that worked for the two whole numbers still holds and Lucy can see this because the calculation is error-free. Too often students' "pattern spotting" is spoilt by errors in the data. If the pattern did not hold, going through the history on the home screen enables a good check for errors. Trying a negative number raises other questions. Lucy had expected the answer –13 for her next example:

-3 + 2 = -1, -1 x 5 = -5, subtract – 12 gives –7

So far, this problem has been used to support Lucy in two aspects of the difficult transition from arithmetic to algebra: changing her view of equality and expanding the range of numbers with which she is comfortable. Now we tackle a third aspect of the transition. Arithmetic can be carried out successfully step by step. It deals with operations taken one after the other. Algebra instead deals with operations in combination, not one step at a time. The step-by-step instructions have to be converted to one expression, expressed in a standard form: *[(number +2)x5 – 4 x number]*. The difficulty of this step is not just a notational issue; not just getting the brackets right. It also requires a conceptual re-orientation affecting performance in algebra and arithmetic throughout secondary school, which many students currently do not master. Home screen calculations support this well.

Students like Lucy will quickly make the conjecture that the finishing number is ten more than the starting number. Lucy was convinced after only two examples. Why not leave it there? I feel strongly that this type of task must not be left at the pattern spotting stage. Even if a thousand numbers have been checked and all have worked, a mathematics lesson ought to go further and seek a general reason for the result. In doing so, a number of important algebraic ideas, such as the distributive property of multiplication over addition can be consolidated. Students ought to be able to see that the ten extra comes from 5 times the 2 added at the start and the original number comes from 5 times the number minus 4 times the number. A test of understanding the structure at this informal level is whether students can make up multi-step formulas to reach other similar results. Stacey and MacGregor (1997a, 1997b) give further information on the transitions that students have to make as they move from arithmetic to algebra.

This example has been built around a very modest little problem to illustrate how much mathematical thinking can be developed using only a very modest aspect of a graphical calculator. However, developing the mathematical thinking requires going beyond the numerical examples that the calculator provides.

Example 2. Searching for reasons, not just fitting data.

The second example is from senior secondary mathematics and a worksheet by John Dowsey and David Tynan (1996) of our Melbourne team is attached. Data has been collected using a temperature probe in a classroom – a very simple procedure with a graphics calculator accessory. The worksheet gives the temperature (T) of a saucepan of water at intervals of 30 seconds for 5 minutes (t is time elapsed). The problem is to predict the temperature later, say after 30 minutes.

This is an impressive lesson, bringing real data into the mathematics class with a convenience hardly imaginable just a few years ago. The data in the tables can be quickly plotted and examined. Then, using the built-in regression facilities of a graphics calculator, students can quickly calculate the regression curves and relevant statistics. The value of r-squared shown on the calculator is given after the equation below. It is an indicator of how close the curve is to the data points.

Linear	T = 93.98 – 5.72t	(r-squared = 0.961)
Quadratic	T = 0.80t^2 – 9.72t + 96.98	(r-squared = 0.997)
Cubic	T = 97.55 – 11.45t + 1.76 t^2 – 0.13 t^3	(r-squared = 0.999)
Exponential	T = 94.52 (0.93)^t	(r-squared = 0.978)

Plotting the regression lines over the data and also examining the r-squared values shows that these all fit well. (See the graphs on the attached worksheet). The cubic polynomial, in particular, is almost exact.[i] However, zooming out on the graphs to predict the temperature after 30 minutes shows that they are all useless. The quadratic predicts that the water boils, the other three graphs predict that it freezes, fast or slowly. This shows the weakness of mathematical analysis that is driven only by the data and not by reasoning. Just as easy access to numbers in Example 1 above can encourage pattern spotting without reasoning, so easy access to data handling can encourage priority to empirical approaches.

Easy access to the standard models on the menu can easily lead to too much data collection and calculation and not enough thinking. We teachers know Newton's Law of cooling predicts exponential drop in temperature, so the exponential regression option is seductive, especially when it fits the data points so well. However, good modelling proceeds on the basis of selecting a function for a reason, not just because it fits the data. In this case, the quantity that decays exponentially is the difference between the temperature of the water (T) and the ambient temperature (A). A model with predictive power can be found by finding the exponential regression of t against (T-A). In this case, the r-squared value is "only" 0.982, not as good a fit to the data as the quadratic or cubic, but a far better model.

This problem exploits the opportunities provided by new technology well, to give real meaning to mathematical ideas. However, it also demonstrates that even practical modelling such as this should not be data-driven, but reason–driven.

Issue to debate

I have offered these two examples firstly to illustrate the usefulness of graphics calculators for teaching mathematics. A graphical calculator enhances the first activity because:

- A wide range of numbers can be easily tried (as with any calculator)
- The home screen records results and facilitates checking
- Students can move conceptually from the step-by-step orientation to using expressions combining operations.

The second activity relies completely on the computational power of the graphical calculator though it is enhanced because:

- It shows the data in two representations, as numbers and plotted points

- It is easy to move between the table of data, the plotted points, the graphs and their symbolic rules
- The regression algorithms and statistics are in-built and accessible

With superficial use, activities that make good use of calculators can encourage a very empirical approach to mathematics. In the first activity, students will quickly spot a pattern (you add ten) , the question is regarded as "done" and the class moves onto the next question, without any of the rich algebra learning experiences that can also be drawn out from situations like this. In the second activity, the data can be plotted, the regression curves can be seen to fit the data well (indeed extremely well) and the question is regarded as "done". But, as explained above, fitting the data is not the most important criterion for deciding whether the model is a good one.

A very empirical approach to mathematics is superficially appealing – it is easier for students to just observe rather than to explain why. More students can participate more easily. However, I believe that justification (maybe not formal proof) is the essence of mathematics and we need to guard against empirical approaches that are advancing into curricula.

My questions to the VC participants are:

- Do you agree that the empirical approach is growing in Mathematics, encouraged in part by increasing access to calculators?
- How can this trend to empiricism be kept in check and justification be promoted?
- What sort of activities exploit the power of advanced software and calculators yet promote justification?
- What arrangements for teaching, curriculum or assessment exploit the power of advanced software or calculators yet promote justification?

References

Dowsey, J. & Tynan, D. (1996*) VCE Graphics Calculator Project (Victorian Board of Studies) Workshop Sample Tasks*. Melbourne: University of Melbourne.

Stacey, Kaye & MacGregor, Mollie (1997a) Building foundations for algebra. *Mathematics in the Middle School.* 2(4, February), 253 – 260.

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Stacey, Kaye (2000 - in press) Problem Solving Approaches and Modelling: Opportunities and Issues. *Proceedings of the AAMT Graphics Calculator Conference.* Sydney, March 30,31, 2000.

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[i] Polynomials are excellent for approximating data. One reason why the cubic is excellent here is that there are four degrees of freedom (a,b,c,d) available in T = a +bt + c t^2 +d t^3. In contrast the default exponential T = a.b^t has only two degrees of freedom.