GOALS FOR A CAS-ACTIVE SENIOR MATHEMATICS CURRICULUM

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This paper canvasses options for goals of using computer algebra systems (CAS) in senior secondary mathematics subjects. CAS may be used in order to align school mathematics with the use of technology in the modern world, or to make students better users of mathematics, or to achieve deeper learning, or to change views of mathematics, or to provide curriculum space to introduce new topics, or some combination of these. The features of CAS making these feasible goals are outlined. To stimulate debate, we propose priorities amongst the goals and propose the relative likelihoods of the enabling mechanisms working in practice.

Introduction

This paper outlines some preliminary issues being addressed by a new study which aims to investigate the changes that regular access to CAS (computer algebra system) supercalculators will have on senior mathematics subjects and the associated assessment in Victoria, Australia. It will explore the feasibility of offering new mathematics subjects that use CAS extensively. In Victoria, schools provide Year 11 and 12 subjects for the Victorian Certificate of Education (VCE) under guidelines in a "Study Design" provided by the Victorian Board of Studies. Final grades, which are used for various purposes including selection to tertiary courses, are derived from a mix of tasks set at the school and external examinations. Subject to the continuing approval of the Board, the curriculum, teaching methods and formal assessment undertaken by students in three project schools will be altered, culminating with the trial in 2002 of an alternative VCE Study Design and examinations using CAS.

The study is funded from 2000–2002 by the Australian Research Grant Strategic Partnerships with Industry Scheme. The Chief Investigators of the project are Gary Asp, Helen Chick, Barry McCrae and Kaye Stacey from the University of Melbourne with David Leigh-Lancaster from the Board as a partner investigator. There are four industry partners: the Board and three calculator suppliers and manufacturers, Hewlett-Packard, Shriro (Casio) and Texas Instruments. The industry partners will supply CAS supercalculators to students in three schools for a three year program of classroom based research. All the schools will use CAS technology that is hand-held. Although this was not the original intention, it means that the broader issues of learning and assessing with full desk or laptop computer capabilities do not have to be addressed. Further details of the project are available from the authors and are given in Stacey, McCrae, Chick, Asp and Leigh-Lancaster (submitted), which also canvasses preliminary decisions about the role to be played by CAS in the formal year 12 assessment.

At this point in history, we believe that the most significant stimulus for change in senior school mathematics is new technology generally and, for the next few years specifically, the advent of affordable computer algebra systems (including symbolic algebra, graphics, statistics, calculus, matrices etc). CAS is acting as an agent of educational change in two different ways simultaneously: it necessitates some change and it provides an opportunity or stimulus for other change (see Figure 1). At least in the long term, the ubiquity of mathematically able software will necessitate change in the mathematical methods that are taught. Responsiveness to new technology may be

Deleted: In the long term, mathematically able software (currently CAS) slow but it will occur Finding good (i.e. simple, quick, routine, widely applicable etc) methods of calculation is critical for all branches of Mathematics. Calculation in a broad sense is a major obstacle to using mathematics and so significant new technology for performing routine procedures (whether abacuses, logarithm tables, slide rules, calculators or computers) will always be embraced for appropriate problems in the long term. CAS therefore necessitates changes to mathematical methods taught for solving problems, but it does not dictate a timetable for change.

Relationship of new technology to curriculum change
CAS necessitates change of methods taught (possibly slowly). CAS provides an opportunity for other change.

Figure 1. CAS changes curriculum and teaching

On the other hand, the arrival of CAS provides an opportunity and a stimulus to move curriculum and teaching in certain desirable directions. These directions can accord with our values and beliefs about what mathematics is or should be (at this level and for these students), tempered by previous experience of what can be achieved in practice. However, exactly which directions for curriculum change are feasible depends on a practical assessment of the actual features likely to be available on affordable machines within a medium timeframe. In this way, there must be an interplay between the changes in senior mathematics that we may desire on the basis of beliefs about mathematics and good mathematics teaching on the one hand, and technical features of the new technology which is underpinning the change, on the other. In this paper we will present, for debate, the major goals that we think can be achieved by using CAS (Figure 2) and relate these to the features of CAS and teaching with CAS which make them feasible (Figure 3). This sets out a framework (Figure 4) that links the goals and mechanisms for achieving them.

In whose interest is curriculum change? A caution

In this paper, we present our preliminary thinking on one of the major policy issues to be resolved through the project: the way in which the curriculum will be adapted to use CAS. This is likely to be a highly contentious issue and so careful appraisal of the evidence relevant to various positions will be needed to guide policy. The main groups involved in this change are the students, the teachers, the curriculum authorities (in this case, the Board), the calculator industry and mathematics education researchers. There are clear benefits for some stakeholders. It is easy to see how the adoption of CAS will benefit the calculator industry. It also benefits researchers, because perturbation in the learning environment provides us with new ways of exploring fundamental issues about teaching and learning mathematics. Leading teachers will also benefit, because their professional lives are enriched by new challenges. We must be certain, however, that students will also benefit from change: both as individuals and as future citizens of a country which aspires to take a place in a competitive global economy based on technology.

Goals for a New Curriculum

Why should CAS be used in senior mathematics, or, indeed, why should it not be used? And, when the goals are clear, how can CAS be used to achieve them? As noted above, deciding on the goals for introducing a new CAS-active mathematics subject requires consideration of both philosophies of mathematics teaching and technical features of CAS. In Figure 2, the goals that we have for the potential new mathematics subject are presented. Note that these goals are additional to the "normal" goals of a mathematics subject at Year 12. They are presented in a proposed order of priority: this order is not yet settled, but is presented as a point to debate. In fact, much of what follows is presented in this spirit, as preliminary thinking to be refined and then operationalised in the light of experiences of teaching with CAS, its impact on students and discussions with other stakeholders.

Goals for CAS-active mathematics
(in priority order – for debate)

1. To make students better users of mathematics
2. To increase congruence between real maths and school maths
3. To achieve deeper learning by students
4. To promote a less procedural view of mathematics
5. To introduce new topics into the curriculum

Figure 2. Goals for a CAS mathematics subject, presented in order of importance

Each of the goals listed in Figure 2 is shorthand for a complex of ideas. Goal 2, increasing congruence, is based on the principle that schools should prepare future citizens for a technologically infused society and that school mathematics should prepare them for mathematics as practised beyond school. In this working world, analysts such as Pea (1993) see intelligence itself not solely as a function of one person's mind but as distributed between members of a working team, technology and symbol systems. Successful professionals form a partnership with "intelligent technology" (Saloman, Perkins and Globerson, 1991), not expecting to be separated from it. Those who agree with this goal may nevertheless differ in their view of what "real world" mathematics is like: is it the "real world" of the mathematician or the engineer, what is the "real world" expert use of by-hand procedures etc? Moreover, an immediate pragmatic concern of teachers is that before taking up their long-term role in the workplace, most senior mathematics students undertake tertiary education and the transition to CAS there is still unclear—although as Galbraith, Haines and Pemberton (1999) note, incorporation of CAS is taking place at an increasing rate. In summary, if a general principle of alignment to the technology used in the "real world" is accepted, there are still many different views about what this might mean in practice.

Everyone would agree with the first and third goals, to make students better users of mathematics and to help them learn better, but again there will be debate on the nature of "better". Do we wish students to be able to solve a greater range of problems or do we want them to be able to solve problems in the current range more reliably? Can a CAS-active mathematics subject help them become more discerning and confident users of mathematics? And what is better learning? Is it more thorough learning of the same topics, or learning deeper properties? Does it involve being able to apply learning to an expanded range of unfamiliar problems? Becoming better users of mathematics will surely require relatively more emphasis on formulating problems in mathematical terms and interpreting results, and relatively less emphasis on methods of solving. Experience has shown, however, that formulating is a difficult step for students and the routine aspects of solving are easier—if formulating cannot be taught effectively, what will be left for average students to do?

The fourth goal relates to the values and beliefs about mathematics as a subject that students (and teachers) might hold. It is a commonly shared belief that Mathematics should be more of a "thinking subject" which encourages higher-order thinking and has less emphasis on performing routine procedures in over-rehearsed standard responses to exercises. School mathematics should mirror genuine mathematical activity and include both mathematical discovery and mathematical modelling. Students need the opportunity to "do mathematics", as well as to practise it. Whilst there is often general agreement on the goal of moving away from routine work, there can be quite different images of what this might mean. Again, the relative importance of proof (or at least justification) compared with a more empirical approach seems a key variable.

The final goal of introducing new topics into the curriculum must be raised as a possibility. If some routine procedures are to be omitted from a curriculum, should not something be put in its place? What might this be? Writing at the beginning of CAS technology on personal computers, Stacey and Stacey (1983) predicted that time saved on calculus techniques by using CAS might be better spent on studying the mathematics for computer science. Is this today's choice? Alternatively, can the time be used to teach the higher order skills of formulating and interpreting more successfully than in the past?

The intention of this section has been to flesh out some aspects of the breadth of each of the five goals listed in Figure 2, and to demonstrate some of the variation in what they may mean, so as to stimulate debate on what is most worthwhile. Amongst our team, there is support for each of goals 1, 2 and 3 as first priority.

Features of CAS enabling the achievement of new goals

As noted above, the goals for CAS-active mathematics cannot be arrived at without assessing the capabilities of the CAS systems that will be available to students in the next few years and the ways in which they can be used in classrooms. If CAS is to be the stimulus of change, then the direction of possible change is to some extent dictated by features of the technology. Fortunately, there is now a growing set of research reports that provide evidence about the likely effects of CAS use.

In Figure 3, we list (again welcoming debate) the major mechanisms by which using CAS can achieve the goals outlined earlier. Again, as a point of debate amongst our team and with others, we have tentatively put the list in order: this time in the order of likely widespread success. In passing, note that some mechanisms (i.e. 1, 2, 3, 4, 6) are a consequence of functional use of CAS (its capacity to produce answers) and others (1, 4, 5, 6, 7) are a consequence of pedagogical use of CAS (using it as a teaching tool). This distinction between functional and pedagogical use is outlined, with examples for basic calculators, by Etlinger (1974).

The first feature listed, providing multiple and easily linked representations of mathematical ideas, is extremely important and well known, featuring in almost all CAS curricula. It is both a functional and pedagogical feature. Not only does it enable students to solve problems by moving between representations (e.g to find a zero of a function graphically), but it provides the teacher with convenient capacity for supporting visualisation in Mathematics, including dynamic classroom demonstrations. Examples include

- how a family of functions changes as a parameter changes,
- how a secant approaches a tangent,
- how changing outliers affect a line of best fit,
- how an optimum value in a linear programming problem varies with parameters.

Such demonstrations can also be made available to students on their own machines for use whenever they want.

Mechanisms by which CAS use might help achieve new curriculum goals (in order of likely success)
1. By providing multiple, easily-linked representations of ideas
2. By freeing up curriculum time (for re-allocation)
3. By enabling less constrained real problem solving
4. By reducing curriculum emphasis on routine procedures
5. By providing "trainer wheels" for learners
6. By providing more options for teaching and problem solving
7. By promoting positive learning strategies

Figure 3. Mechanisms by which CAS use can achieve curriculum goals.

There are, of course, unanswered questions about the use of multiple representations, especially whether students beginning in an area will be able to learn well across a range of representations and what choices teachers will make. Kendal and Stacey (1999) have some preliminary results. They studied the different choices of representation that teachers of introductory calculus made for teaching and how this impacted on students learning. Because CAS provides technological support for working numerically, graphically and symbolically, a range of methods (e.g. for finding a tangent) which were previously only available in theory (e.g. because they involve excessive calculations) become available in practice. There are more options for teaching and there are more options for problem solving (see also Tynan, Stacey, Asp and Dowsey, 1995). Kendal and Stacey found that teachers will choose to highlight attributes of CAS which support their own beliefs and values about mathematics. Teachers who value routine procedures can find on a CAS a plethora of routine procedures to teach students; teachers who value insight can find many ways in which they can demonstrate links between ideas better than ever before.

Many experimenters assume using CAS will free-up curriculum time so that time that is currently spent on less-valued objectives will be able to be reallocated to more valued objectives. Time is freed up in several ways. Firstly, some lesson time is saved when tasks which are presently done by hand, are sometimes done by CAS. For example, students can draw a lot of graphs quite quickly, freeing up time for class discussion. Secondly, some curriculum re-sequencing studies, which Heid (1997) reports have had positive results for calculus, operate on the principle that CAS enables concepts to be taught in advance of skills. This makes the later teaching of by-hand skills more efficient because they are learned on a sound conceptual footing. McCrae, Asp and Kendal (1999) report an Australian study on this principle. They decided to spend an allocated 20 lessons for introductory calculus by first building conceptual understanding very carefully. Ten lessons focussed on the concept of a derivative before differentiation methods were treated with much reduced practice time. The results of these Year 11 students were comparable with those of Year 12 students in the same school. In the initial concept building, however, graphical and physical (datalogging) tools were more useful than the symbolic capability of CAS.

Drijvers (1999) took a different approach to time re-allocation. He outlines two units taught in the Netherlands that featured imaginative problems requiring optimisation of complicated functions in real life situations. Time previously spent on practising by-hand skills was re-allocated to early CAS-supported investigations, from which students could see the purpose of the mathematics to be learned. Drijvers' experiment is also an example of how CAS enables less constrained problem solving.

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More controversially, time can be freed up by not teaching certain by-hand skills at all. This is the option which will need the most careful consideration of costs and benefits, but which also offers the greatest potential for really creating space in the curriculum. We are yet to be convinced that teaching students to use a CAS will not absorb whatever additional time is created by other mechanisms. One important cost of not teaching certain skills is that students generally feel uneasy about using entirely "black-box" methods (at least at this stage of the change). Drijvers (1999) showed this, as have Australian studies. In the McCrae et al (1999) study, 15 of the 59 students made at least one written comment during the study that expressed some uneasiness or concern. Pierce (1999) working with tertiary students also finds this uneasiness, even though her students have unconstrained access to CAS in the examinations. For example, 13 students undertaking an introductory calculus unit were asked if they thought CAS would offer fresh hope to students with difficulties in mathematics and 6 expressed some sort of uneasiness including:

> Student 7: "No, I think you need to know what the computer is working out and how."
>
> Student 8: "CAS offers the student the answers without really needing the basics. I personally would have enjoyed more time spent on the basics."

"Trainer wheels" studies are designed around the theory that CAS use can support learners as they learn to carry out mathematical procedures for themselves. This can reduce cognitive load and enable students to recover from errors within an attempted solution. Tynan and Asp (1998) worked with younger students beginning algebra (48 Year 9 students at one school in two classes). They used CAS for two reasons: firstly as "trainer wheels" when students were learning to solve equations by the "do the same to both sides" method. Students could use the CAS to test the effect of actions (such as adding $2x$ to both sides), to check each line of their work and to recover when they had made mistakes. CAS was also used to show students the power of algebraic methods of solving problems, before they could reliably solve one variable linear equations by hand. All too often, students learn how to solve equations, but when faced with a problem situation they either do not realise that equation solving is relevant, or they feel they are unlikely to be successful with an equation, and so they resort to numerical methods or guess and check (Stacey & MacGregor, 1999). Tynan and Asp reported clear success in getting students to appreciate the power of algebraic methods. For example, on one item, 56% of student taught with CAS set up an equation and tried to solve it using algebraic "do the same to both sides" methods. In the class taught traditionally, only 26% did this.

There is now a significant body of research that supports the proposal that use of CAS can promote positive learning strategies, such as exploration of ideas, group work, discussion and negotiation of meaning. Pierce (1999), for example, provides evidence that these strategies are more prevalent in her CAS classes. Positive learning strategies are not, however, specific to computer algebra systems and may be more a consequence of computer use where there are shared computers and public displays.

How CAS can assist in achieving new goals

Figure 4 highlights for us the type of assumptions about CAS use that we make when we claim that CAS will enable us to better achieve certain objectives. It also provides guidelines for the way in which CAS will need to be built into a new curriculum and its teaching, learning and assessment.

Goals	CAS-related mechanisms for achievement
To make students better users of mathematics	By enabling less constrained real problem solving By freeing up curriculum time By providing multiple, easily-linked representations of ideas By providing more options for teaching and problem solving
To achieve congruence between real maths and school maths	*Use of CAS in teaching and assessment* By enabling less constrained real problem solving
To achieve deeper learning by students	By promoting positive learning strategies By providing multiple, easily-linked representations of ideas By freeing up curriculum time By providing "trainer wheels" for learners
To promote a less procedural view of mathematics	By reducing curriculum emphasis on routine procedures By enabling less constrained real problem solving
To introduce new topics into the curriculum	By freeing up curriculum time

Figure 4. Goals for CAS-active mathematics and how they might be achieved.

There is not a simple correspondence between the goals of Figure 2 and the mechanisms for achieving these goals in Figure 3. In fact, several mechanisms will operate to achieve each goal. Figure 4, again for discussion and debate, sets out the main links. For example, Figure 4 shows how the first goal is feasible because CAS can reduce the constraints on the problems that can be considered at school (e.g. there is no longer such a need to have "nice" functions and numbers), curriculum time can be freed to re-allocate to more experience of problem solving, and multiple representations make more methods of solving problems feasible. The second goal of achieving congruence between mathematics at school and mathematics as it is used in "real life" (in the case of senior mathematics, in scientific and technical employment) is largely achieved simply by the presence and use of CAS in teaching and, importantly, in assessment. We assume that complicated algebraic calculation, accurate graphing, statistical calculations, etc will be carried out by machine in virtually all workplaces, supplemented by a fair degree of mental or by-hand skill. Since CAS presence is not really a mechanism, it is in italics in Figure 4. Figure 4 similarly shows the features of CAS use on which achievement of the other three goals depends.

Conclusion

This paper has outlined options for the goals of introducing CAS into senior secondary mathematics. CAS is seen as driving some change in some directions and providing an opportunity for other change. There is a range of goals that might be achieved and our project must prioritise them. The achievement of any of the new goals depends on functional aspects of CAS and pedagogical choices about CAS use. The paper has highlighted both the options and the choices to be made, and the connections between them.

Acknowledgment

We wish to thank the members of the Melbourne TAME group, especially Lynda Ball, Helen Chick, Peter Flynn, Margaret Kendal and Robyn Pierce, for their helpful comments.

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