TOWARDS A VCE MATHEMATICS SUBJECT WHICH ACTIVELY USES CAS

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This paper describes preliminary work to investigate use of calculators equipped with a computer algebra system (CAS) in Victorian Certificate of Education mathematics subjects. A new VCE subject, parallel to Mathematical Methods, but actively using CAS calculators, may possibly become available from 2004. The new technology will be used to enhance learning and to widen student options for problem solving and some of the ways in which this could happen are illustrated. Changes to curriculum will be necessary and there will be associated changes to assessment.

Introduction

For thirty years, new information technology has been a major force driving change in mathematics curriculum and assessment. In the 1960s the introduction of electric calculators changed the way that arithmetic was done in business. As a consequence, the goals of the mathematics curriculum needed to be thoroughly revised. Later, when hand-held electronic calculators became affordable for students at school, curriculum and assessment began to change at all levels — a change involving such a fundamental re-assessment of goals and teaching methods that it continues today. Adapting to the technology of the four-function calculator was, for example, a major theme of the 1998 NCTM Yearbook (Morrow & Kenney, 1998). School systems around Australia and worldwide are now seriously grappling with the use of graphics calculators in senior school mathematics teaching, curriculum and assessment. However, we see this as a transient stage, a prelude to the next phase of technology-driven challenges and opportunities for teaching mathematics, which will centre on computer algebra systems.

Computer algebra systems (CAS) can perform all the routine procedures of mathematics normally covered in secondary school and university, including drawing graphs, calculating with vectors and matrices, and doing algebra, calculus and statistics. These systems have existed for over twenty years, even on small systems such as home and school computers, but until the recent advent of hand-held machines of moderate cost, they have had little effect on school mathematics. Wilf, in his 1982 article evocatively entitled "The Disk with the College Education", looked forward to the day when students might have a $39.95 pocket calculator with a long LCD window for display and lots of buttons such as DERIV (for differentiation) and MATRIX, performing operations from the major areas of senior secondary and early tertiary mathematics.

Although the $39.95 version has not yet literally come, CAS calculators may soon be priced such that all senior mathematics students can own one. By the time of publication of this paper, we expect that some hand held CAS calculators will be priced at about $AUD160, comparable with the price of the best graphics calculators, while computer based CAS software site licenses are becoming increasingly affordable. There are a number of sites with information about CAS software. Two such sites can be found at http://www.sciface.com and http://www.softmath.com/index.html.

As the availability of CAS in schools increases, so does the need to produce appropriate policy responses.

Researching Possible Curriculum Change

This paper outlines some preliminary issues to be addressed in a study funded from 2000–2002 by the Australian Research Grant Strategic Partnerships with Industry Scheme. The study aims to investigate the changes that regular access to CAS calculators may have on senior mathematics subjects and to explore the feasibility of offering new subjects that use CAS extensively. The Chief Investigators of the project are Gary Asp, Helen Chick, Barry McCrae and Kaye Stacey from the University of Melbourne, with David Leigh-Lancaster from the Board of Studies as a partner investigator. The four industry partners are the Board, and three calculator suppliers and manufacturers: Hewlett-Packard, Shriro (Casio) and Texas Instruments. The industry partners have supplied CAS calculators to students in three volunteer schools for a three-year program of classroom-based research. With the cooperation of the Board, the content and formal assessment undertaken by these students for Year 12 would be altered, possibly culminating with the trial in 2002 of a VCE subject using CAS as an alternative to the current *Mathematical Methods 3/4* subject. The major features are summarised in *Figure 1*. Actual implementation of this requires the Board's continuing approval, which will be informed by the findings of the project in earlier years.

For each project school: CAS support from one industry partner		
2000	**2001**	**2002**
Mathematical Methods 1/2	Mathematical Methods 3/4 Assessment: Standard VCE + trial CAS paper	
	Mathematical Methods 1/2	New CAS Mathematics 3/4 Assessment: New Board CAS paper

Figure1. Proposed time-line of research and assessment changes for each project school.

For the calculator industry partners, the project enables their products to be tested rigorously within an Australian curriculum context, investigating the suitability of the products' capabilities, interfaces, notations and physical characteristics. Materials for training teachers to use the CAS calculators will be enriched by experiences in the project schools. For the Board, the project will result in advice to support the development of policy for curriculum and assessment, covering issues such as:

- which parts of Year 12 assessment could permit CAS, forbid it or require it;
- the protocols for the use of CAS calculators in assessment;
- how (and if) examination questions can be set to be fair to users of CAS calculators of different brands and models that have different capabilities (the involvement of different industry partners is critical for this);
- subsequent redevelopment of lower secondary mathematics curricula, and suggestions for post-secondary mathematics and mathematics-related studies.

Beyond these practical issues, the scientific importance of the study lies in its advancement of our detailed knowledge of how students learn mathematics and how it can best be taught. This new learning environment provides us with new opportunities for research into the teaching of mathematics. When students have CAS in class and in examinations, the need for memorising routine procedures may be enormously reduced, yet the need for conceptual and structural understanding is almost certainly undiminished and possibly increased. Effective policy development (in particular for assessment used for university selection) requires research, especially as the possible changes are likely to be substantial and subject to robust debate. We know of no similar study being undertaken anywhere in the world. Thus, the results of this research are likely to have implications for many education systems.

The next section shows some possible ways that CAS could be incorporated into the teaching and learning of trigonometric functions in the current *Mathematical Methods*. We then present an application task, requiring definition of functions, which is an adaptation of *Bushwalking with Kim* from the Mathematics Study Design (Board of Studies, 1999). These two examples demonstrate how using CAS requires a good grasp of algebra, that it provides a way of enabling students to investigate mathematical situations before making generalisations, and that it enables students to attempt algebraically some questions that they may have attempted graphically before. Following this we show how CAS could be used in current coursework for *Mathematical Methods* and finally, we outline preliminary thinking about the needed changes to assessment.

Using CAS in work on circular functions

Currently *Mathematical Methods* students are required to graph and differentiate trigonometric functions, and find solutions to trigonometric equations. The use of CAS expands the possibilities for teaching and learning in this topic by enabling students to explore trigonometric functions, further develop rules for themselves and investigate more complex problems than those possible by paper and pencil calculations. The following examples give an indication of some possible applications of CAS in teaching this topic.

Solutions of trigonometric equations

With a graphics calculator, students can find approximate solutions for trigonometric equations within a restricted domain. A CAS calculator enables students to solve trigonometric equations graphically and algebraically, and to explore how changing particular values in an equation can affect the solutions.

Consider solving $2\sin 3x = 1$ for $0 \le x \le 2\pi$. This example is relatively straightforward and students can easily find that the first solution for this problem is $x = \frac{\pi}{18}$. Using a graphics calculator without CAS capabilities, it is possible to determine approximate solutions for this equation. One such value is shown in *Figure 2*.

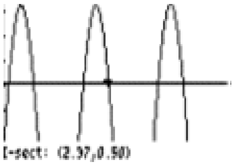


Figure 2: Graph of Y1=sin(3x) and Y2=$\frac{1}{2}$

The CAS calculator, however, will give exact solutions for this trigonometric equation: $x = -\frac{12\pi n1 - 5\pi}{18}$ and $x = \frac{12\pi n1 + \pi}{18}$ (see *Figure 3*). It is essential for students to understand what this output actually means. They need to understand that there are two families of solutions, and the 6 solutions in the restricted domain can be obtained by substituting particular integer values for the parameter *n1* into the general expressions. For example, if *n1*=0 then $x = \frac{5\pi}{18}$ and $x = \frac{\pi}{18}$. See *Figure 4* for the first solution. Substituting selected values for *n1* gives the other four solutions $\frac{29\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$, and $\frac{25\pi}{18}$.

```
RAD XYZ HEX R= 'X'        ALG
{HOME}
:SOLVE(2·SIN(3·X)=1,X)
{X=- 12·π·n1-5·π  X= 12·π·n1
         18              18
{X=-((12*π*n1-5*π)/18
),X=(12*π*n1+π)/18}
```

Figure 3: General solution for *2sin(3x)=1*

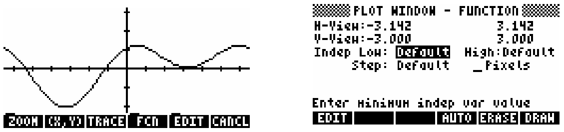
```
RAD XYZ HEX R= 'X'        ALG
{HOME}
: 12·π·0+5·π
       18
                          5·π
                          18
```

Figure 4: One solution for *2sin(3x)=1*

CAS does solve the equation algebraically, but students need an understanding of the multiple solutions of trigonometric equations and of *n1* as a parameter, so that they can interpret the result obtained.

Applications of trigonometric functions

CAS calculators deal easily with complicated functions and therefore enable more applications to be undertaken. For example, students could be asked to draw a graph of the position of a particle given by $f(x) = \sin x + \cos 2x$ at time x for $-\pi \le x \le \pi$, find the velocity function, and then the speed when $x = \frac{\pi}{6}$. The graph of $f(x)$ and the window used are shown in *Figure 5*.



```
PLOT WINDOW - FUNCTION
H-View:-3.142             3.142
V-View:-3.000             3.000
Indep Low: Default  High:Default
     Step: Default   _Pixels

Enter minimum indep var value
```

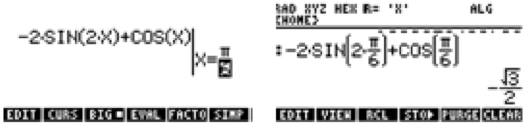
Figure 5: Graph and viewing window for *sin(x) + cos(2x)*

To find the velocity function, students can use the CAS differentiation to give *–2 sin(2x)+cos(x)* (see *Figure 6*).

```
:DERIV(SIN(X)+COS(2·X),X)
          -(2·SIN(2·X))+COS(X)
```

Figure 6: Derivative of *sin(x) + cos(2x)*

To find the velocity when $x = \frac{\pi}{6}$, the expression can be evaluated at $x = \frac{\pi}{6}$ using CAS to give a result of $-\frac{\sqrt{3}}{2}$ as in *Figure 7*. So the speed of the particle would be $\frac{\sqrt{3}}{2}$ or 0.87 m/s to two decimal places.



```
-2·SIN(2·X)+COS(X)|X=π/6
```

```
:-2·SIN(2·π/6)+COS(π/6)
                    -√3/2
```

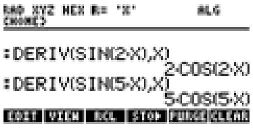
Figure 7: Calculation of derivative at $x = \frac{\pi}{6}$

Inductive development of rules for differentiation

Rules such as the chain rule can be developed inductively using CAS. Students can use CAS to produce the derivative for a number of cases, look for patterns in their results and conjecturing the rule in their own words. *Figure 8* shows the output when *sin(3x^2)* is differentiated. Students could produce a number of results similar to this and using their knowledge of differentiation of polynomials, they can predict the chain rule. A further example, also leading to an instance of the chain rule for composite functions, would be to explore the derivative of *sin(kx)*, as shown in *Figure 9*. Alternatively, using CAS, students could differentiate the general expression *sin(kx)* to obtain *kcos(kx).*

```
             6·X·COS(3·X^2)
:COS(3·X^2)·3·2·X
             6·X·COS(3·X^2)
```

Figure 8: Derivative of *sin(3x^2)*



```
:DERIV(SIN(2·X),X)
                 2·COS(2·X)
:DERIV(SIN(5·X),X)
                 5·COS(5·X)
```

Figure 9: Differentiating *sin(2x)* and *sin(5x)*

The use of CAS supports this inductive thinking and discovery. It is an extremely powerful tool in a classroom situation that allows students to explore mathematical patterns to develop rules and then ask whether they are generally true and why.

Using CAS to work with general functions

Using a CAS calculator (or computer software) will enable students to adopt a more algebraic approach to some questions. Functions with algebraic variables to represent all changeable values, including parameters related to given conditions for a problem situation, can be defined. This can be a stimulus for students to consider all changeable values in a problem situation without immediately assigning numerical values, as they have to do using a graphics calculator. Solutions can be found by using the CAS to symbolically manipulate the function as needed, and then by allocating values to explore how the changes affect the solution. Through using CAS, students are learning to define and operate on functions. They can easily move from the symbolic representation to graphs or tables, and generate families of related graphs.

The problem below is adapted from the *Bushwalking with Kim* Analysis Task found in the current Mathematics Study Design (Board of Studies, 1999, p. 152). It illustrates how students using CAS can:

- consider the problem statement and make sense of it;
- determine the changeable values and assign appropriate variable names;
- use mathematical understanding to find the information to define functions;
- define functions in terms of the changeable values;
- assign specific values to some changeable values;

- symbolically manipulate functions and solve equations linked to functions and others derived from them to find a solution;
- draw graphs and/or use tables to confirm symbolic solutions; and
- change assigned values to explore aspects of the problem under varying conditions and interpret the effect of the changed conditions.

Bushwalking with Kim

Kim wishes to walk from A to B. The direct route of 14 km, will take her through rugged bush country. However, there is a large square clearing situated as shown in the diagram. This clearing has one diagonal along the perpendicular bisector of the direct route and one corner, C, at the midpoint of the direct route. Kim believes that time will be saved if she travels from A to B on a route passing through P and Q, where PQ is parallel to the direct route. The side-length of the square is 7 km. Kim can travel at *b* km/h in bushland and *c* km/h in the clearing. Choosing reasonable values for *b* and *c*, find and describe the route for which her travelling time will be least and compare it with the direct one.

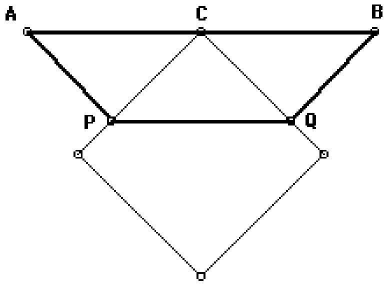


Diagram for problem statement

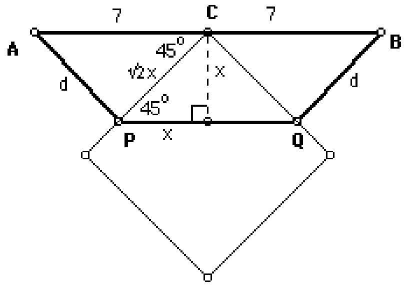
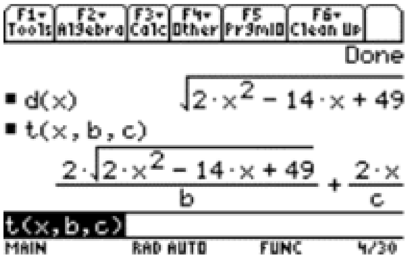


Diagram with assigned variable names and found information making it possible to define functions

Define the *time function t* as ***t(x, b, c)* = 2*x/c* + 2*d(x)/b***. Using the Cosine Rule, define $d(x) = \sqrt{\left(\sqrt{2}x\right)^2 + 7^2 - 2\sqrt{2}x \times 7 \times \cos 45^\circ}$. Entering *d(x)* again shows how *d(x)* is defined, but this time the CAS has simplified the defining expression. Entering *t*(*x*,*b*,*c*) now gives *t(x,b,c)* using the defined function for *d(x)* as shown at the right in *Figure 10.*



Done

■ d(x) $\sqrt{2\cdot x^2 - 14\cdot x + 49}$

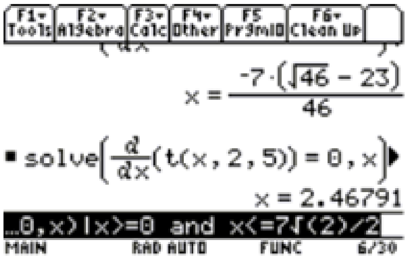
■ t(x, b, c)

$$\frac{2\cdot\sqrt{2\cdot x^2 - 14\cdot x + 49}}{b} + \frac{2\cdot x}{c}$$

t(x,b,c)

Figure 10: Defined function *d(x)* used in *t(x, b, c)*

The command Solve (*d/dx*(*t*(*x*,2,5)) = 0, *x*) | 0 $\le x$ and $x \le \frac{7\sqrt{2}}{2}$ gives stationary points for *b* = 2 km/h and *c* = 5 km/h. The solution is given in terms of the square root of 46 in exact form and then in approximate form as 2.46791 (See *Figure 11*). The restriction on the domain can still be seen on the entry line of the calculator.



$$x = \frac{-7\cdot(\sqrt{46} - 23)}{46}$$

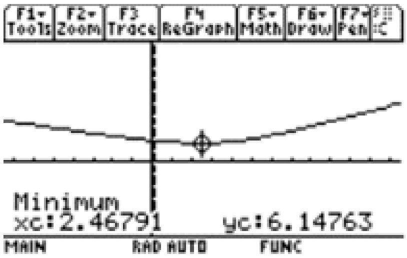
■ $\text{solve}\left(\frac{d}{dx}(t(x, 2, 5)) = 0, x\right)$ ▸

$x = 2.46791$

…0,x)|x>=0 and x<=7√(2)/2

Figure 11: Solution when *b*=2km/h and *c*=5 km/h

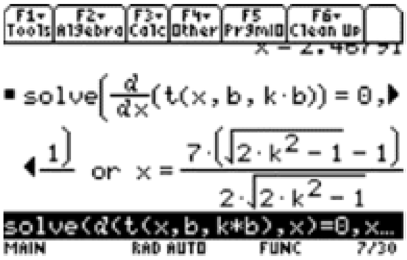
Setting *y1(x)* = *t*(*x*,2,5), we can produce the graph of the *time function* and the related table. Using the find minimum feature we can find the minimum for *y1*(*x*) to confirm the result obtained symbolically. See *Figure 12*. Consulting the table first helps to find the values for the lower and upper bounds for determining the minimum using this feature. Additional functions for different choices of *b* and *c* can be easily entered and graphed as desired.



```
Minimum
xc:2.46791       yc:6.14763
```

Figure 12: Minimum value for *y1(x)*

We can now give a solution for *x* in terms of the scale factor *k* such that *c* = *k*b* as shown in *Figure 13*. The calculator provides two possible solutions and even alerts us to the possibility that one is questionable. The solution shown is the correct one. Why is another solution produced?



$$\blacksquare\ \text{solve}\left(\frac{d}{dx}(t(x,b,k\cdot b))=0,\ \ldots \quad \ldots 1) \text{ or } x=\frac{7\cdot\left(\sqrt{2\cdot k^2-1}-1\right)}{2\cdot\sqrt{2\cdot k^2-1}}$$

```
solve(d(t(x,b,k*b),x)=0,x…
```

Figure 13: Solution for *x* in terms of *k*.

We now have the optimum value of *x* as a function of *k*. Values can be assigned to *k* to determine how the optimum *x* changes with *k*. For example:

- What happens when *k* =1?
- What is the route when Kim can travel twice as fast in the clearing as in bushland, fives times as fast, etc.

We can also choose a distance for *x* and find the value of k needed so that travelling this distance across the clearing results in the minimum travel time for the entire route.

Using CAS in current coursework assessment

CAS can be used in each of the tasks for coursework assessment in current VCE mathematics subjects - tests, analysis tasks and application tasks. The facility of CAS to readily move between numerical, graphical and symbolic modes of operation, as well as use both exact and approximate representations of numbers, enables students to develop work on assessment tasks that teachers can readily link to the demonstration of key knowledge and skills related to the use of technology.

The effective use of CAS requires good symbol sense and a sound understanding of the concepts of variable and function. The ability of CAS to define general functions in terms of key parameters, as demonstrated above, supports mathematical analysis. This in turn can lead to students developing arguments to establish general propositions, or proof. For example, students know that graphs of quadratic functions have reflection symmetry. Graphically, this often amounts to visual affirmation of the symmetry of an exhibited graph in a vertical line *x* = *k*. This may also be demonstrated by substituting *x* values equally spaced either side of *x* = *k* into the rule for a particular quadratic function. Perhaps some students would have followed this through to the next step, where, for a general quadratic function $f(x) = ax^2 + bx + c$, it is shown that $-b/2a = k$ and

$$f\left(\frac{-b}{2a}-h\right)=f\left(\frac{-b}{2a}+h\right) \text{ for } h\in R$$

Is there a similar general symmetry property for cubic functions? The basic cubic function with rule $f(x) = x^3$ has rotational symmetry by $\boldsymbol{R_\pi}$ about the origin, because $f(-x) = -f(x)$. This is also true for cubic functions with rules of theform $f(x) = ax(x^2 - b)$, which can readily be verified algebraically and checked visually. A more general question is whether *any* cubic function has this type of rotational symmetry about some suitable point on the curve of its graph. The use of CAS can assist all students, and not just the most algebraically able, to analyse this question and develop an in-depth understanding of cubic functions and their graphs. If a function has rotational symmetry about a point (X_0, Y_0), then $f(X_0 - h) - Y_0 = -[f(X_0 + h) - Y_0]$ for all values of h: for x values at equal distances from the point of symmetry, the function is equally above and below the point of symmetry. Students would need to make a conjecture about what (X_0, Y_0), might actually be. Many may guess that it is a point of inflection and then be able to demonstrate the symmetry by substituting the value ($X_0 = -b/3a$) into the equations above and simplifying with the CAS. Development of these ideas could be used as a basis for an analysis task in the current *Mathematical Methods Unit 4*.

Mathematics Assessment with (or without?) CAS

In this section, we canvass how a CAS active mathematics subject might be assessed. McCrae (1996) found that access to graphics calculators would impact on only 6% of the 1994 VCE *Specialist Mathematics* CAT 2, but that about 60% would be affected by the availability of CAS. This suggests that a VCE examination that is CAS active would require changes in question styles.

The situation elsewhere

There are some countries worldwide that have policies permitting the use of substantial CAS in national examinations. In Denmark, CAS will soon be permitted in all mathematics examinations for 15-19 year old students. In France, any calculator, including those with CAS, is permitted in examinations, though the examinations are intended to be CAS neutral. In Austria and parts of Germany, teachers can decide whether CAS is permitted in examinations. Many calculators with and without CAS capabilities, including the three being used in our project (Casio FX 2.0, HP 49G & TI-89), are allowed to be used by students in the USA College Board for Advanced Placement (AP) *Calculus AB & BC* examinations (http://www.collegeboard.org/ap/calculus/html/exam001.html). Computers, non-graphing scientific calculators, devices with a QWERTY keyboard (eg. TI-92), pocket organisers, and electronic writing pads (eg. Palm) are not allowed.

For one part of the examination in AP Calculus, students are expected to have a calculator with the capability to produce the graph of a function, find the zeros of a function, compute the derivative of a function numerically, and compute definite integrals numerically. If students use one of these four features in an examination question, then they are required to write down the set up of the problem and can then state the result from the calculator. If another capability of a calculator is used then students must show the mathematical steps in the solution of the problem and not just write down the calculator result. In the second part of the examination, students are not permitted to use any calculator. The AP examiners' restrictions are designed to maintain an acceptable level of equity, whilst encouraging students to use advanced technology that is expensive (so that they have adequate computational tools for the examination), and to allow a 'breathing space' for teachers to incorporate the new technology into their teaching. However, the Development Committee for AP Calculus has stated that it will 'continue to monitor the developments of technology and will reassess the testing policy regularly' (http://www.collegeboard.org/ap/calculus/html/exam002.html).

Is there a need for CAS free assessment?

VCE mathematics examinations have in recent years moved from being graphics calculator free, to graphics calculator neutral, to graphics calculator active. This means that the solutions of some questions are greatly facilitated by the use of a graphics calculator. This transition was a natural progression as students, teachers and the wider educational community recognised the important role that graphics calculators can play in the learning and teaching of mathematics and as their cost dropped. Similarly, since the use of CAS will be an integral part of the learning and teaching in the proposed new subject, it is essential that this be reflected in assessment.

There was little debate as to whether a component of the external assessment of the current *Mathematical Methods* should remain graphics calculator free, despite this option being adopted elsewhere. In VCE Mathematics examinations assessment access to an approved graphics calculator is assumed. The impact that CAS availability would have on current examinations, as noted earlier, requires careful consideration when determining the nature of the examinations for the new subject.

Stacey, McCrae, Chick, Asp & Leigh-Lancaster (2000) have identified four conditions that would have to be met in the implementation of a course that uses CAS extensively and that has CAS active examinations, namely that:

- students would not be disadvantaged in tertiary courses that use different CAS or that do not use CAS;
- brand neutral CAS active assessment is possible;
- students have adequate calculation skills and algebraic insight to use CAS; and
- that 'true understanding' can be tested at least as well in a CAS active examination as in a CAS free examination.

In a CAS-active course there will still be algebraic manipulative skills that students should be able to perform mentally or by-hand. Like Herget, Heugl, Kutzler and Lehmann (2000), we believe that these essential by-hand calculation skills may be relatively low-level, though this will vary from person to person, as it does now with arithmetic. A student will not be able to use CAS effectively otherwise. A student would, for example, have to be able to see the equivalence of $-(x-y)/x$ with $y/x-1$ to use CAS. Several of the problems above illustrate more significant examples – the appearance of parameters in the general solution of the trigonometric equation, the need to define a function in *Bushwalking with Kim,* and understanding how the symmetry of the parabola or cubic can be expressed algebraically. We envisage that the algebraic insight and understanding required by students to use CAS may well be higher than that which is currently expected or demonstrated.

What by-hand skills should be practised to efficiency? One option could be that students in the new subject should be able to solve linear equations by hand, but need not know the formula for solving a quadratic equation; they should be able to differentiate a polynomial or negative power and know that the derivative of a trigonometric function is also trigonometric, but may not need to know the exact derivative or the product rule. However, when we consider the current situation we see that 30% of students could not select the derivative of $x(x^2-3x-9)$ from a list of five possibilities on the 1999 *Mathematical Methods 3/4* CAT 2 (Board of Studies, 2000, p. 11). The main reason, however, for requiring mastery of some procedures is that they will be essential for effective CAS use and so will be tested *implicitly* on a CAS-active examination. Our initial position is that they will not also need to be tested explicitly in a CAS free examination, but this is a question that will be researched.

An argument for CAS-free assessment is the contribution that practice at *extended mathematical reasoning* is assumed to make to general intellectual development, especially 'logical thinking'. So, although it may not be important to test whether students can perform isolated techniques, it may be important to test whether they can plan and carry out a *sequence* of such techniques for a desired purpose as in a mathematical proof. Again referring to 1999 *Mathematical Methods 3/4* CAT 3, the average mark for Question 4c, which required students to find the exact minimum value of $\sin(\pi x/10)-\cos(\pi x/10)+3$, was 0.99 out of 5 (Board of Studies, 2000, p. 15). Of course this question is really only a routine application (requiring five main steps: finding a somewhat complex derivative, equating it to zero, solving the equation, finding the function value and checking it gives a minimum) but this is the degree of 'extended mathematical reasoning' on a *Mathematical Methods* examination. We believe that extended reasoning can also be tested in a CAS-active examination, but this requires further research.

As noted above, our initial position is that examinations for the new subject should be CAS active. This potential future mainstream VCE mathematics subject will target a wide cohort of students, including 'users' of mathematics and students who will go on and be more than just 'users' of mathematics. We agree with Buchberger (1990, p.11) that 'for math users it might be really sufficient in the future that they learn to formulate their mathematical problems and to use a good symbolic math software system for obtaining solutions'. We see in CAS use an opportunity to free up some time to devote to the goal of formulating mathematical problems and so achieve this successfully with more students than at present. As Buchberger points out, though, learning basic techniques gives students more insight into the relevant area of mathematics. Consequently, it will be important to consider the respective functions of *Specialist Mathematics* and the proposed new course and possibly take the opportunity to include more algebra and proof in *Specialist Mathematics*(with consequent implications 'down the school').

Conclusion

Extending the range of technology used in school mathematics is a long-term process. Through current coursework requirements, teachers can begin to experiment with having students use CAS, thereby developing their own familiarity with the possibilities of the new technology and also giving students other approaches for solving problems. Even limited use of CAS in school mathematics problems quickly highlights, for teachers and for students, the need for algebraic insight, for understanding the nature of solutions and of problems, and for being able to plan a solution. Teachers will also find that, like graphics calculators, CAS technology can be used pedagogically, scaffolding students as they undertake unfamiliar processes and supporting a wide range of classroom demonstrations. We hope that our research project will enable the construction of a strong CAS alternative to *Mathematical Methods* that will build on the good work that has been going on with graphics calculators in Victorian schools.

References

Board of Studies. (1999). *Mathematics Study Design.* Melbourne: Publisher as author.

Board of Studies. (2000). VCE Report for teachers 1999 Mathematics Cycle 3. Melbourne: Publisher as author.

Buchberger, B. (1990). Should students learn integration rules. *SIGSAM Bulletin* 24, 91(1), 10-17.

Herget, W., Heugl, H., Kutzler, B., & Lehmann, E. (2000). *Indispensable manual calculation skills in a CAS environment.* Accessed June 13, 2000: http://www.kutzler.com/article/art_indi/indisp.htm

McCrae, B. (1996). The use of calculators in Victorian Certificate of Education examinations: Looking ahead. *Australian Senior Mathematics Journal 10*(1), 65-71.

Morrow, L.J. & Kenney, M.J. (1998) *The teaching and learning of algorithms in school mathematics* (1998 Yearbook). Reston, VA; National Council of Teachers of Mathematics.

Stacey, K., McCrae, B., Chick, H., Asp, G., & Leigh-Lancaster, D. (2000). Research-led policy change for technologically-active senior mathematics assessment. In J. Bana and A. Chapman (Eds.), *Mathematics Education Beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, pp. 572-579). Fremantle: MERGA.

USA College Board. (2000). *Advanced Placement (AP) Calculus*. Accessed August 22, 2000: http://www.collegeboard.org/ap/calculus/index.html

Wilf, H.S. (1982). The disk with the college education. *American Mathematical Monthly,* 89, 4-8.