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Communication and Learning in Small Group Discussion.

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Abstract

Discussion in small groups is widely recognised to be a potentially powerful method of learning mathematics. It is therefore important to understand the elements of communication which make it effective. In this study, small group discussion to reduce misconceptions about division was analysed. Although groups in which effective learning took place appeared on the surface to engage with the task in a similar way to groups where learning did not take place, close examination revealed substantial differences in interaction patterns and stark differences in the mathematical substance of the discussion. Effective learning is associated with highly interactive behaviour, where children engage fully with the mathematical content of the task and respond to each others' ideas. Teachers who are alerted to the characteristics of effective group discussion may be able to help children make it a better learning tool.

Introduction

Learning in small groups is now a popular teaching methodology in elementary school classrooms. A variety of co-operative, group learning schemes have been advocated to achieve both social and academic goals and to address issues of classroom management, such as mixed-ability teaching. In mathematics, only a minority of studies have shown a statistically significant difference between achievement in the various co-operative schemes and whole class or individual learning. However, where significant differences have been found they have usually favoured the co-operative schemes (Davidson and Kroll, 1991). This indicates that the factors which make peer interaction a valuable learning device are not automatically present when the classroom is organised around learning in groups. Many teachers will choose to organise their classrooms around group learning in

order to achieve social or management goals. If these teachers are aware of the factors which effect academic outcomes, then learning outcomes also may be enhanced.

A series of studies have examined the internal dynamics of cooperative groups which are associated with successful learning. Members of groups which learn effectively interact more with each other, interact more with the task and utilise more cognitive strategies than ineffective groups (Sharan and Shachar, 1988; Webb, 1991). Webb (1991) reviewed 17 studies of mathematics learning which examined verbal aspects of peer interaction. She reported that elaboration of help given was a critical feature of the peer interaction. Students who gave high-level elaboration (for example, by explaining to their team mates and giving extended answers) showed higher achievement than those who did not. Webb used partial correlations to support her claim that the high-level elaboration caused the high achievement, rather than the alternative possibility that initially high achievement made the elaboration possible.

For many students, learning in mathematics is plagued by persistent misconceptions which are often untouched by traditional teaching methods. As yet, published research on group learning has only studied the acquisition of new knowledge. However, it seems likely to us that reduction of misconceptions may be one of the most useful functions of group learning in mathematics, because of the potential for the group situation to promote and then resolve cognitive conflict. Bell (1986), for example, has developed the "diagnostic teaching" methodology which uses tasks designed specifically for groups to reduce misconceptions through small group and class discussion. In this study, we analyse children's interaction during a card-placing activity for small groups which Bell found to effectively reduce misconceptions (when supplemented by class discussion). Tasks of this nature have not been investigated in a process-outcome study.

Therefore the purpose of this study is to compare the patterns of interaction associated with effective reduction of misconceptions with the patterns found to be effective in other studies which have focussed on acquisition of new information. In addition, this study also documents more directly than previous studies how the effectiveness of the discussion relates to the subject matter content of the children's talk. Striking differences are found.

Method

Choice of task and misconceptions addressed.

Concepts related to division were chosen as the content of the discussion because the difficulties are widespread across the age range from ten years to adulthood, persistent, important to address and well documented (Ball, 1990; Hart, 1981; Tirosh and Graeber, 1990). Bell (1986) refined a task to help children learn about division of a smaller number by a larger number and sort out the confusions that arise from the different order used in the two notations, the lunar sign $)$ and Rahn's sign \div . In the task, a group of children have to place 36 cards in appropriate places on a grid drawn on a rectangular board. Figure 1 shows the correctly completed placement of cards on the board. There are three pairs of division questions: one question of each pair requires a larger number to be divided by a smaller number (e.g. $6)12$ and $12\div 6$) and the other question requires the much harder reverse situation with the same numbers (e.g. $12)6$ and $6\div 12$). In this paper, the former divisions will be called larger/smaller divisions and the latter divisions will be called smaller/larger divisions.

Procedure

Groups of four children in Grades 5 and 6 were withdrawn and videotaped working on the activity. Children were presented with the board, which had headings and a selection of the cards (those in boldface type in Figure 1) already in place. When the board was complete, they were asked to compare their board with one which was nearly correct and said to have been organised by another school. This gave them an opportunity to correct their own board and discuss any remaining misconceptions.

The videotapes, including all of the discourse and all gestures which indicated the order of division operations, were transcribed. A modified version of Sharan and Shachar's (1988) scheme was found to be the most satisfactory, because it focused on aspects of the interaction and not on linguistic form. The unit of analysis was the "turn talking". The second scheme coded the content of children's discourse in six categories which are described below. A pre-test was administered one week before the activity and the post-test was administered three weeks after it. Coded discourse was then related to the results of the tests.

EXAMPLE	WORDS	\div	ANS	$\overline{) \quad}$	ANS
8 apples are shared between 2 boys. How many apples does each boy get?	8 divided by 2	$8 \div 2$	4	$\overline{2)8}$	4
2 apples are shared amongst 8 girls. How much apple does each girl get?	2 divided by 8	$2 \div 8$	1/4	$\overline{8)2}$	1/4
You have \$12. Each present costs \$6. How many presents can you buy?	12 divided by 6	$12 \div 6$	2	$\overline{6)12}$	2
What is 6 divided by 12?	6 divided by 12	$6 \div 12$	1/2	$\overline{12)6}$	1/2
4 kilometres split into 1/2 kilometre sections. How many sections are there?	4 divided by 1/2	$4 \div 1/2$	8	$\overline{1/2)4}$	8
1/2 a kilometre split into 4 sections. How long is each section?	1/2 divided by 4	$1/2 \div 4$	1/8	$\overline{4)1/2}$	1/8

FIGURE 1. The completed board showing all 36 cards correctly placed (from Bell,1986).

Results

Pre-test and post-test results

The task was effective in promoting learning, as half the class showed improvement on the difficult smaller/larger divisions at the post-test after three weeks. Full details of the improvement are given in Gooding and Stacey (to appear). In both tests, there were nine questions, of which the two smaller/larger divisions were the target questions. Almost all students were correct on the larger/smaller divisions on both occasions. On the pre-test only 4 of the 28 children got either of the two target questions correct. On the post-test, 15 were correct on one target item and 10 were correct on the other. Individual students who improved their score on the target questions are designated "improvers". The others are designated "non-improvers", except for the one child who had all questions right on both tests.

Effective and ineffective groups

Five groups were designated as effective because their members increased their score on the two target questions by an average of half a question or more. The other two groups (one of boys, one of girls) were designated as "ineffective". The learning outcomes for individuals were mixed. In each of the effective groups, there were at least two improvers. In every group, there was at least one non-improver. One of the ineffective groups had completed the board correctly, one incorrectly. It is interesting that the only group in which no-one made gains contained the only member of the class who could calculate smaller/larger items correctly from the beginning. He did not explain to others and they placed cards with the least discussion.

Interactive aspects of discussion

The two ineffective groups had students taking the fewest number of turns talking, 174 turns and 297 turns respectively. In contrast the number of turns talking in effective groups ranged from 362 to 820 turns. Amongst the effective groups, there is no direct correlation between number of turns talking and amount of improvement.

The results of coding the transcripts with the modified version of the Sharan and Shachar (1988) scheme are shown in Table 1, where the mean percentage of turns talking in each category are given for both individuals and for groups. The children in the ineffective groups interacted less than those in the effective groups. This is so even though the use of percentages in Table 1 adjusts for the very marked differences in the amount of talk between groups. The lower level of interaction is

reflected in the greater percentage of talk in the thinking aloud category for ineffective groups and the generally lower percentages in the categories, such as responding, which indicate intellectual interaction. Similar observations hold when the comparison is between individuals (improvers and non-improvers) rather than groups, although a little less strongly.

TABLE 1. Percentages of turns talking for each interaction type for groups and individuals.

	Effective Groups N=5 N=2		Ineffective Groups N=14 N=13		Improvers	Non-Improvers
<u>Asking questions</u>						
of a previous speaker	2.4	3.0	2.2	3.0		
from own thinking or working	6.6	7.7	5.7			
reading a word problem	1.0	0.0	1.2	0.4		
Subtotal	10.5	9.6		11.1		9.1
<u>Responding</u>						
to a request for clarification		5.5	6.2	5.6		
agreeing	9.4	5.5	9.2	8.2		
disagreeing	13.9	11.6	13.2	14.2		
repeating	4.5	0.5	4.7	2.6		
Subtotal	33.9	23.1	33.3	30.6		
<u>Directing</u>	5.8	2.7	4.9	6.3		
<u>Explaining with evidence</u>	7.9	4.8	8.0	6.5		
<u>'Thinking aloud' when reading or placing the cards</u>	36.0	55.6	36.2	43.6		
<u>Proposing ideas</u>	1.4	0.7	1.7	0.6		
<u>Commenting (affective)</u>	1.6	2.3	1.7	1.7		
<u>Refocusing discussion</u>	2.7	1.4	3.1	1.4		

Effective groups read the questions on the "example" cards out loud whereas members of the ineffective groups did not do this at all (see Table 1). Members of effective groups and improvers from all groups gave more explanations with evidence and repeated each other's statements more frequently. This is a way in which children agree, possibly reflecting on each other's answer at the same time. (Noddings, 1985) Because the percentage figures given in Table 1 adjust for the

differing amounts of talk in the groups, the absolute numbers of explanations and repetitions are very much greater for effective learners.

Mathematical content of the discussion

In addition to the interactive discourse analysis above, the content characteristics of the discussion were analysed. Turns talking were classified (where relevant) as correct or incorrect smaller/larger division statements, correct larger/smaller divisions, correct and incorrect discussion of the order of division and statements generally identifying non-commutativity of division. There is no category of incorrect larger/smaller divisions because there were very few of these.

There was almost no explicit mathematical discussion in the ineffective groups. Neither of the ineffective groups specifically discussed the order of the division operation for either division sign, whereas the effective groups discussed this explicitly 8 or more times each. The ineffective groups each made only 5 calculations of any sort. Each of the effective groups made at least 27 calculations. Table 2 gives the medians of the number of instances identified in the talk of each group. The number of instances of talk in each category for the two ineffective groups was, with only one exception, always less than the corresponding number of instances for each of the effective groups. The probability that the low results of the ineffective groups could be attributed to chance was calculated by the Mann-Whitney test to be less than 5% ($n_1=5$, $n_2=2$, $U = 10$, $p<0.05$). At the individual rather than the group level, the results are similar.

Discussion

Interactive aspects

The broad patterns of interaction which have been associated with higher achievement in previous studies have also been found in this new setting. Members of effective groups interacted more. Generally they helped each other more by responding and explaining to a greater extent during the task. Improvers engaged in the activity by working out the division problems in the task explicitly. They used more specific mathematical talk than non-improvers for every category. The coding of mathematical content and the coding of interaction patterns have affirmed in this new setting Webb's (1989) proposition that giving help at a high level of elaboration of help to others is related to achievement.

TABLE 2. Average number of statements in specific categories of mathematical talk (Standard deviations in brackets)

	Correct smaller/larger	Incorrect smaller/larger	Correct larger/smaller	Identifying non-commutativity	Correct discussion of order	Incorrect discussion of order
Groups						
Medians for effective groups	7.0	17.0	8.0	10.0	10.0	6.0
Medians for ineffective groups	0.5	4.0	0.5	0.5	0.0	0.0
Significance of difference	*	*	*	*	*	n.s.
Individuals						
Means for improvers	3.4	4.4	3.1	3.5	3.9	1.1
Standard deviations	(3.18)	(4.37)	(3.87)	(2.77)	(4.30)	(1.12)
Means for non-improvers	0.5	2.9	0.8	0.8	0.2	0.6
Standard deviations	(1.59)	(3.17)	(1.48)	(1.31)	(0.42)	(1.00)
Value of t (d.f. =25)	2.84	1.00	1.90	3.07	3.08	1.16
Significance of difference	**	n.s.	n.s.	**	**	n.s.

* significant at the 5% level, ** significant at the 1% level, n.s. not significant

Background knowledge and achievement

Although the numbers of students are too small to draw a statistically tested conclusion, it is perhaps significant that all the non-improvers had got both questions wrong on the pre-test. It is likely that a threshold of background knowledge was required to gain benefit from participation in the task. However, there is no simple relationship between the background knowledge of group members (as measured by the pre-test) and improvement. Group 7 contained the only student who had completely correct answers on the pre and post tests, yet the others in the group did not improve. On the other hand, group 4 was effective, but the answers they gave on the pre-test showed not even a primitive "does not go" awareness of order of division. Further investigation, including detailed mapping of the prior conceptual understanding of students, is required here.

Thinking aloud

The most noticeable difference from the frequency of codings given in Sharan and Shachar's study (1988) was the very much higher percentage of unstructured ideas coded as 'thinking aloud' (3% , compared to an average of 41% over all groups here). Rather than explaining this difference by modifications made to the coding scheme, we propose that this reflects differences in the nature and difficulty of the task. The children in this study spent a greater percentage of their time 'thinking aloud' because they were working to understand difficult ideas. The difficulty of the

task is indicated by the fact that all groups, especially the effective groups, frequently made mistakes. 'Thinking aloud' is similar to Pimm's (1987) notion of stating a problem over and over. Pimm considered that this can help to establish a mental image, access to which is necessary for a solution. He sees it as a positive, clarifying aspect of problem solving. Our data associates it with ineffective learning, perhaps because children who were less able to cope with the task were unable to purposely exchange mathematical ideas and explain them to each other clearly. Students' capabilities and background knowledge influence their behaviour and learning in a group task.

Conclusion

The task proved highly effective for reducing a persistent error, and most impressively after three weeks, for half of the class in less than half an hour. Children in effective groups were found to engage in mathematical discussion in a specific way. Additionally, although the setting was different in important respects to the settings used in other studies of peer learning, effective discussion shared many of the features found in other studies. Both those groups which were effective and those individuals who improved, interacted more than the others and they engaged at a higher level of elaboration. Ross and Raphael (1990) have suggested that relatively unstructured tasks may be the most effective for achieving higher level cognitive objectives. This may be one feature which makes Bell's task appropriate for the reduction of misconceptions.

The analysis of the mathematical content of the discussion showed substantial differences in engagement with the central ideas of the task between effective and ineffective groups. In summary, effective groups:

- Talked more, with more mathematical content (some of which was wrong).
- Explicitly discussed the central idea (order of division).
- Worked together by reading the questions on the cards 'out loud' and repeating each others' statements.
- Proposed ideas, gave explanations with evidence and refocused discussion more often.
- Responded to the questions of others more.

Improvers made many mistakes in their mathematical talk. They were able to learn even though they made mistakes and heard others make mistakes. We hypothesise that Bell's task succeeded in creating cognitive conflict, bringing their

misconceptions out into the open where the children in effective groups could grapple with them. This mechanism will be investigated in a future study. A detailed analysis of background knowledge and conceptual development of group members, errors made during discussion and the resulting learning could illuminate the process of learning and reducing misconceptions through discussion substantially.

Children's discussion is widely considered to be important for learning, yet the mechanisms which make it effective are still not well understood. However, the stark differences found here in the degree to which groups interacted with the mathematical content of the discussion call for clear action. Teachers can be advised to draw their students' attention to the need for engagement with the task (e.g. by actively calculating answers to the problems given, not passively trying to match cards), for student discussion of each others' calculations and for discussion of reasons for placement of the cards during the activity. At the very least, teachers can use knowledge of such characteristics of effective groups to identify and intervene with groups which are unlikely to be learning. In this study, careful analysis of the ways in which effective and ineffective groups interacted revealed substantial differences, but the magnitude of these differences had not been grasped as the experimenter watched the groups interacting. This points to the need for specific teacher education about these important factors.

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