

CHARACTERISTICS OF SMALL GROUP DISCUSSION REDUCING MISCONCEPTIONS

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Abstract

This pilot study establishes characteristics of dialogue during a group discussion designed to reduce misconceptions related to division. Groups of children were videotaped and transcripts were coded for mathematical and interactive aspects of the discourse. Although there were important differences between the nature and purpose of this task and those that have been used in previous studies of peer learning, once again a highly interactive pattern of discourse was found to be associated with effective learning. Substantial differences were found in the ways in which effective and ineffective groups engaged with the content of the discussion. Teachers who are alerted to the characteristics of effective group discussion may be able to help children make it a better learning tool.

CHARACTERISTICS OF SMALL GROUP DISCUSSION REDUCING MISCONCEPTIONS

The extensive research on various forms of peer learning in small groups has established group discussion as an effective method of learning (Johnson, Maruyama, Johnson and Nelson, 1981; Slavin, 1989; Sharan and Shachar, 1988). Although a minority of the studies in Mathematics have shown a statistically significant difference between achievement in the various cooperative schemes and whole class or individual learning, where significant differences have been found they have usually favoured the cooperative schemes (Davidson and Kroll, 1991). A second strand of research has focused on the internal dynamics of cooperative groups (Johnson and Johnson, 1985) and on the 'process-outcome' relationship (Webb, 1985) in order to understand the mechanisms which can make cooperative learning more successful. These peer learning studies have been concerned with the acquisition of new knowledge, not the equally important instructional goal of elimination of persistent misconceptions. The present study examines the dialogue of small groups which are using discussion to eliminate misconceptions.

Members of groups which learn effectively interact more with each other, interact more with the task and utilise more cognitive strategies than ineffective groups (Sharan and Shachar, 1988; Tingle and Good, 1990; Webb, 1989). Webb (1989) reviewed 17 studies of mathematics learning which examined verbal aspects of peer interaction. She reported that the degree of elaboration of help given was a critical feature of the peer interaction. Students who gave high-level elaboration (for example, by explaining to their team mates and giving extended answers) showed higher achievement than those who did not. Webb used partial correlations to support her claim that the high-level elaboration caused the high

achievement, rather than that high achievement made the elaboration possible. Receiving elaborated responses is a more complex matter, not necessarily related learning (Vedder, 1985).

The consistent finding that high-level elaboration is related to learning indicates that an analysis of the subject matter content of the talk may be informative. Surprisingly, this has not been examined in previous discourse analysis studies. Therefore this paper analyses the connection not only between subsequent learning and interaction but also between the content of the group discussion and learning. Children's discussion was videotaped. Data obtained from transcripts was coded for both interactive and content aspects and linked to test results.

For many students, learning in mathematics is plagued by persistent misconceptions, which are often untouched by traditional teaching methods. Bell (1986) addressed this problem through his "diagnostic teaching" methodology which uses tasks designed specifically for groups to reduce misconceptions through small group and class discussion. In this study, we analyse children's interaction during a card-placing activity for small groups which Bell found effective. Group tasks designed to reduce misconceptions have not been investigated in a process-outcome study.

There are several other salient features of the task which serve to locate it in studies of small group learning. There is no reward for the group other than the intrinsic pleasure of successful completion (c.f. Slavin 1980). There is no written component; all of the time is spent in discussion of one task, which in this study took from 10 to 25 minutes. This is longer than in other studies. For example, using data provided for the studies in Webb's (1989) survey, we estimate that in the study by Peterson and Swing (1985), answer checking in small groups occurred for an average of three minutes per session, whilst Lindnow, Wilkinson and Peterson (1985) reported discussion lasting from two to ten minutes. Finally, this task is much less structured

than other co-operative teaching techniques which have been studied. The absence of role assignment may be one of the characteristics of the task which make it suitable for reducing misconceptions, since both Slavin (1980) and Ross and Raphael (1990) have noted that less structured co-operative techniques are especially suitable for higher level cognitive learning. Ross and Raphael (1990) suggest that highly structured cooperative techniques possibly reduce communication, motivation and the extent to which help is given.

In summary, the purpose of this study is to compare the patterns of interaction of groups where learning takes place with the patterns established by other studies which have used tasks of a rather different nature and with a different purpose. In addition, it seeks to document how the effectiveness of the discussion relates to the subject matter content of the children's talk. The task studied here fits in with any classroom organisation and, as we show, is quite effective in reducing persistent misconceptions. It is important to document behaviour which makes such a practically important teaching methodology effective. Teachers who are alerted to characteristics of effective behaviour may be able to guide children towards making group discussion more profitable.

Method

Choice of task and misconceptions addressed.

Concepts related to division were chosen as the content of the discussion because the difficulties are widespread across the age range from ten years to adulthood, persistent, important to address and well documented (Ball, 1990; Bell, Greer, Grimison and Mangan, 1989; Hart, 1981; Resnick, Nesher, Leonard, Magone, Omanson and Peled, 1990; Tirosh and Graeber, 1990). Bell (1986) refined a task which has been used

successfully to help children learn the following inter-related conceptions about division:

- (i) that division is not commutative (i.e., that $a \div b$ and $b \div a$ are not interchangeable)
- (ii) that division of a smaller by a larger number is possible (e.g. many children only say 3 divided by 6 "can't be done")
- (iii) that division by a number between zero and one is possible and the result is larger than the number originally divided. (For example, the answer to "4 divided by $\frac{1}{2}$ " is 8, which is larger than 4. Division does not always "make the number smaller".)
- (iv) that the two commonly used notations (the lunar sign $)$ and Rahn's sign \div) work in opposite directions. For example, $3 \div 6$ corresponds to $6)3$ and not to $3)6$. The confusion is also reflected in the associated verbalisations such as "3 divided by 6" and "6 (divided) into 3".

EXAMPLE	WORDS	\div	ANS	$)$	ANS
8 apples are shared between 2 boys. How many apples does each boy get?	8 divided by 2	$8 \div 2$	4	$\overline{2)8}$	4
2 apples are shared amongst 8 girls. How much apple does each girl get?	2 divided by 8	$2 \div 8$	$\frac{1}{4}$	$8)\overline{2}$	$\frac{1}{4}$
You have \$12. Each present costs \$6. How many presents can you buy?	12 divided by 6	$12 \div 6$	2	$\overline{6)12}$	2

What is 6 divided by 12?	6 divided by 12	$6 \div 12$	$\frac{1}{2}$	12)6	$\frac{1}{2}$
4 kms split into $\frac{1}{2}$ km sections. How many sections are there?	4 divided by $\frac{1}{2}$	$4 \div \frac{1}{2}$	8	$\frac{1}{2})4$	8
$\frac{1}{2}$ kilometre split into 4 sections. How long is each section?	$\frac{1}{2}$ divided by 4	$\frac{1}{2} \div 4$	$\frac{1}{8}$	$4)\frac{1}{2}$	$\frac{1}{8}$

Figure 1. The completed board showing all 36 cards correctly placed (from Bell,1986).

In the task, a group of children have to place 36 cards in appropriate places on a grid drawn on a rectangular board. Figure 1 shows the correctly completed placement of cards on the board. There are three pairs of division questions: one question of each pair requires a larger number to be divided by a smaller number (e.g. $6)12$ and $12 \div 6$) and the other question requires the much harder reverse situation with the same numbers (e.g. $12)6$ and $6 \div 12$). In this paper, the former divisions will be called larger/smaller divisions and the latter divisions will be called smaller/larger divisions.

Webb (1989) commented that task structure, which was not investigated at all in her 17 studies, is a factor that has a powerful effect on the incidence of high and low level elaboration in small groups. This task was selected after preliminary trials, which indicated that more elaborated mathematical discussion occurred in this than in other tasks.

Procedure

One composite class of 28 Grade 5 and 6 children (average age 10 and 11 years) was selected. The class had had some experience of group work. Groups of four children were withdrawn and videotaped working on the activity. It was considered that groups of four children would be likely to contribute sufficient information (correct and incorrect) to the discussion of the task to identify and examine misconceptions. The children were assigned to single sex groups to reduce sources of variation in the study, as previous studies have shown giving and receiving help to be related to sexes of requester and provider (Lindow et al, 1985; Webb 1984).

Children were presented with the board, which had headings and a selection of the cards (those in boldface type in Figure 1) already in place. They were requested to talk about what they were doing as much as possible. When the board was complete, they were presented with a second complete board which was nearly correct. Children were only told it had been completed at another school and were asked if they agreed with it. This gave them an opportunity to correct their own board and discuss any remaining misconceptions.

The videotapes, including all of the discourse and all gestures which indicated the order of division operations, were transcribed. Two coding schemes were developed. Firstly, several schemes for coding discourse (Noddings, private communication; Sharan and Shachar, 1988; Sinclair and Coulthard, 1975) were trialled on the transcripts. A modified version of Sharan and Shachar's scheme was found to be the most satisfactory, because it focused on aspects of the interaction and not on linguistic form. The unit of analysis was the "turn talking". Sharan and Shachar's scheme was illustrated with examples in their paper: examples of our adaptation of it are given in Figure 2. The second scheme coded the content of children's discourse in six categories which are described below. In total, the seven

groups produced over three thousand turns talking, which were analysed using the two schemes.

A pre-test was administered one week before the activity and the post-test was administered three weeks after it. The classroom teacher gave the answers to the pre-test immediately afterwards but did not teach this topic. In both the tests of this pilot study, there were nine questions. The two smaller/larger divisions (shown in Table 1) were the target questions for this pilot study. Individual students who improved their score on the target questions are designated "improvers". The others are designated "non-improvers", except for the one child who had all questions right on both tests. Groups were designated as effective if their members increased their score on the target questions by an average of half a question. Other groups were designated as "ineffective". Summary data from the coded transcripts were related to the test results.

Results

Groups were free to take as much time as they wanted over the task and they spent all their time (from 10 to 25 minutes) solidly in discussion. The length of time on task was the time for which card placement accompanying the discussion occurred. When they announced they were finished, Groups 2, 3, and 5 had correctly placed the cards on the board. Groups 1, 4, 6 and 7 had errors in the placement.

Pre-test and post-test results

Learning through the group activity, lasting for the three weeks to the post-test, was generally effective. Table 1 shows the results on the smaller/larger target questions, with results on the easier larger/smaller divisions for comparison. A chi-squared test showed the improvement on

the target questions was highly significant (chi-squared = 11.0, d.f. = 1, $p < 0.001$).

Table 1.
Results on the Target Questions of the Pre-test and Post-test. (N = 28)

	Smaller/larger		Larger/smaller	
Pre-test questions	$6\overline{)3}$	$4 \div 16$	$5\overline{)15}$	$12 \div 3$
Number correct (N = 28)	4	1	25	25
Post-test questions	$6\overline{)3}$	$5 \div 25$	$3\overline{)15}$	$12 \div 4$
Number correct (N = 28)	15	10	22	26

In total, half the class improved on the target questions. All three students who had one of the the target questions correct on the pre-test had both correct on the post-test. Of the 24 children who had none correct on the pre-test, 13 did not improve, 5 made a gain of 1 question and 6 made a gain of 2 questions. The one student who had both target questions correct on the pre-test also had both questions correct on the post-test. This analysis underestimates the amount of learning that occurred because some non-improvers revealed growth in their awareness of the problem, although their final answers were not completely correct. For example, instead of simply not distinguishing $5 \div 25$ from $25 \div 5$, some wrote $5 \div 25$ "does not go".

Effective and ineffective groups

Five groups were considered to be effective, and two groups (one of boys, one of girls) ineffective. In each of the effective groups, there were at

least two improvers. In every group, there was at least one non-improver. One of the ineffective groups had completed the board correctly, one incorrectly. The gains, and the resulting classification of groups, are shown in Table 2.

Table 2.
Improvement on Smaller/Larger Divisions and Amount of Discussion.

Group	Net* Gain	No. of students correct on pre-test		Effective or not	No. of improvers	No. of turns talking
		6)3	4 ÷ 16			
1. boys	+4	0	0	Eff	2	399
2. boys	+4	1	0	Eff	3	362
3. girls	+5	1	0	Eff	3	609
4. girls	+3	0	0	Eff	3	407
5. girls	+1	0	0	Not	1	297
6. boys	+3	1	0	Eff	2	820
7. boys	0	1	1	Not	0	174

* Net gain from pre-test to post-test on the two smaller/larger divisions totalled over all group members.

Table 2 also shows the group membership of students who were correct on the pretest smaller/larger items. It is interesting that the only group in which no-one made gains contained the only member of the class who could calculate smaller/larger items correctly from the beginning. He did not explain to others and they placed cards with the least discussion.

Interactive aspects of discussion

The two ineffective groups had students taking the fewest number of turns talking. Table 2 includes a crude measure of the amount of talk estimated from the number of turns talking that students took on the transcript.

Amongst the effective groups, there is no direct correlation between number of turns talking and improvement.

The results of coding the transcripts with the modified version of the Sharan and Shachar (1988) scheme are shown in Table 3, where the mean percentage of turns talking in each category are given for both individuals and for groups. The categories of the coding scheme, illustrated with examples, are shown in Figure 2.

<u>Asking questions</u> of a previous speaker	<i>How many 1/2's in 4?</i>	
from own thinking or working	<i>Who's got 2? That goes there.</i>	
reading a word problem	<i>1/2 km split into 4 sections. How long is each section? We've got to work that out.</i>	
<u>Responding</u> to a request for clarification	S1: <i>How many 1/2's in 4?</i> S2: <i>There are 4 wholes, right?</i>	
agreeing	<i>Yeah</i>	
disagreeing	<i>You're wrong</i>	
repeating	S3: <i>No, 8 goes there</i> S2: <i>No, 8 goes there</i>	
<u>Directing</u>	<i>Put it down</i>	
<u>Explaining with evidence</u> replacing 1/8 card with 8]	<i>No, 4 how many 1/2s is 8 [4 ÷ 1/2 Pointing left to right</i>	<i>&</i>
<u>'Thinking aloud' when reading or placing the cards</u>	<i>12 divided by 6. All right 12 divided by 6 is there.</i>	
<u>Proposing ideas</u>	<i>It's 2 so they're not the same answer so we can't put them in</i>	
<u>Commenting (affective)</u>	<i>We've finished. Done it.</i>	
<u>Refocusing discussion</u>	<i>Yeah. That's what you're meant to do. You're meant to set it out under this (pointing to</i>	

headings).

Figure 2. Examples of coding scheme categories for interactive aspects.

**Table 3.
Percentages of Turns Talking for Each Interaction Type for Groups and Individuals.**

	Effective Groups N=5		Ineffective Groups N=14		Improvers N=13		Non-Improvers	
<u>Asking questions</u>								
of a previous speaker	2.4		3.0		2.2		3.0	
from own thinking or working	7.1		6.6		7.7		5.7	
reading a word problem	1.0		0.0		1.2		0.4	
Subtotal	10.5		9.6		11.1		9.1	
<u>Responding</u>								
to a request for clarification	6.1		5.5		6.2		5.6	
agreeing	9.4		5.5		9.2		8.2	
disagreeing	13.9		11.6		13.2		14.2	
repeating	4.5		0.5		4.7		2.6	
Subtotal	33.9		23.1		33.3		30.6	
<u>Directing</u>	5.8		2.7		4.9		6.3	
<u>Explaining with evidence</u>	7.9		4.8		8.0		6.5	
<u>'Thinking aloud' when reading or placing the cards</u>	36.0		55.6		36.2		43.6	
<u>Proposing ideas</u>	1.4		0.7		1.7		0.6	
<u>Commenting (affective)</u>	1.6		2.3		1.7		1.7	
<u>Refocusing discussion</u>	2.7		1.4		3.1		1.4	

The children in the ineffective groups interacted less than those in the effective groups. This is so even though the use of percentages in Table 3 adjusts for the very marked differences in the amount of talk that are shown in Table 2. The lower level of interaction is reflected in the greater percentage of talk in the thinking aloud category for ineffective groups and

the generally lower percentages in the categories, such as responding, which indicate intellectual interaction. Similar observations hold when the comparison is between individuals (improvers and non-improvers) rather than groups, although a little less strongly.

Effective groups read the questions on the "example" cards out loud whereas members of the ineffective groups did not do this at all (see Table 3). Members of effective groups and improvers from all groups gave more explanations with evidence and repeated each other's statements more frequently. This is a way in which children agree, possibly reflecting on each other's answer at the same time (Noddings, 1985). Because the percentage figures given in Table 3 adjust for the differing amounts of talk in the groups, the absolute numbers of explanations and repetitions are very much greater for effective learners.

Non-improvers and children in ineffective groups asked questions of others relatively more often than the effective learners did. However, they received relatively fewer responses. A similar observation has been made by Webb (1989). Webb's (1989) proposition that high-level elaboration of help given is related to achievement is also confirmed. High level elaboration of help given corresponds to the categories of responding to requests for clarification, explaining with evidence and proposing ideas in Table 3. These are all higher for effective groups.

Mathematical content of the discussion

In addition to the interactive discourse analysis above, the content characteristics of the discussion were analysed. Turns talking were classified (where relevant) as correct or incorrect smaller/larger division statements, correct larger/smaller divisions, correct and incorrect discussion of the order of division and statements generally identifying non-

commutativity (the fact that order matters for division). There is no category of incorrect larger/smaller divisions because there were very few of these.

There was hardly any explicit mathematical discussion in the ineffective groups. Neither of the ineffective groups specifically discussed the order of division operation for either division sign, whereas the effective groups discussed this central idea explicitly 8 or more times each. The ineffective groups each made only 5 calculations of any sort. Each of the effective groups made at least 27 calculations. Table 4 gives the medians of the number of instances identified in the talk of each group. (Median scores are reported and tested because the distribution is positively skewed.) The number of instances of talk in each category for the two ineffective groups was, with only one exception, always less than the corresponding number of instances for each of the effective groups. The probability that the low results of the ineffective groups could be attributed to chance was calculated by the Mann-Whitney test to be less than 5% ($n_1=5$, $n_2=2$, $U = 10$, $p<0.05$).

Table 4.
Average Number of Statements in Specific Categories of Mathematical Talk (Standard deviations in brackets)

	Correct smaller/larger	Incorrect smaller/larger	Correct larger/smaller	Identifying non-commutativity	Correct discussion of order	Incorrect discussion of order
Groups						
Medians for effective groups	7.0	17.0	8.0	10.0	10.0	6.0
Medians for ineffective groups	0.5	4.0	0.5	0.5	0.0	0.0
Significance of difference	*	*	*	*	*	n.s.
Individuals						
Means for improvers	3.4	4.4	3.1	3.5	3.9	1.1
Standard deviations	(3.18)	(4.37)	(3.87)	(2.77)	(4.30)	(1.12)
Means for non-improvers	0.5	2.9	0.8	0.8	0.2	0.6
Standard deviations	(1.59)	(3.17)	(1.48)	(1.31)	(0.42)	(1.00)
Value of t (d.f. =25)	2.84	1.00	1.90	3.07	3.08	1.16
Significance of difference	**	n.s.	n.s.	**	**	n.s.

* significant at the 5% level, ** significant at the 1% level, n.s. not significant

At the individual rather than the group level, the results are similar. The improvers engaged in the activity by working out the division problems in the task explicitly. They used more specific mathematical talk than non-improvers for every category. Means and standard deviations and the results of t-tests are given in Table 4.

Members of all groups made mistakes calculating smaller/larger divisions. This indicates the degree of difficulty of the task for the children. In fact, both improvers and non-improvers made more incorrect than correct calculations when saying smaller/larger divisions "out loud". Perhaps surprisingly, the improvers made more mistakes with the smaller/larger divisions than did the non-improvers. However, a smaller percentage of their calculations were incorrect (56% for improvers, 85% for non-improvers from Table 4).

It is interesting to note that in group 5 no correct mathematical talk occurred, yet one girl did improve. The transcript shows that she learnt from the correct answers on the board said to be from the other school. Group 3, which made the greatest gains, was the only group which explicitly discussed order of division operations without mistakes. They made other mistakes when dividing but were always clear in their explicit discussions of order.

Discussion

Interactive aspects

Differences have been outlined above between the setting and task investigated in this paper and the settings and tasks of the various peer learning schemes used by previous studies of characteristics of effective

discussion. Many of Sharan and Shachar's (1988) categories which had been developed to code the verbal interactions of cooperative learning History and Geography discussions did apply to this task. The broad patterns of interaction which have been associated with higher achievement in previous studies have also been found in this new setting. Members of effective groups interacted more. Generally they helped each other more by responding and explaining to a greater extent during the task. The coding of mathematical content in Table 4 and the coding of interaction patterns in Table 3 have affirmed Webb's (1989) propositions that high-level elaboration of help given is related to achievement and that not receiving help when it is sought is associated with lack of achievement.

The most noticeable difference from the frequency of codings given in Sharan and Shachar's study (1988) was the very much higher percentage of unstructured ideas coded as 'thinking aloud' (3% , compared to an average of 41% over all groups here). Rather than explaining this difference by modifications made to the coding scheme, we propose that this reflects differences in the nature and difficulty of the task. The children in this study probably spent a greater percentage of their time 'thinking aloud' because they were working to understand difficult ideas. 'Thinking aloud' is similar to Pimm's (1987) notion of stating a problem over and over. Pimm considered that this can help to establish a mental image, access to which is necessary for a solution. He sees it as a positive, clarifying aspect of problem solving. Our data associates it with ineffective learning, perhaps because children who were less able to cope with the task were unable to purposely exchange mathematical ideas and explain them to each other clearly. Students' capabilities and background knowledge influence their behaviour and learning in a group task.

Background knowledge and learning

The task enabled half of the class to reduce their misconceptions about division. However, all effective groups contained individual members who did not improve, despite the specific mathematical nature of the effective groups' discussion. They did not learn simply by being present in effective groups, even though they heard and often engaged in the mathematical discussion themselves.

Although the numbers of students are too small to draw a statistically tested conclusion, it is perhaps significant that all the non-improvers had got both questions wrong on the pre-test. Background knowledge of individuals and of groups collectively is known to be important in learning in the group context (Tingle & Good, 1990, Webb, 1991). It is likely that a threshold of background knowledge was required to gain benefit from participation in the task. However, there is no simple relationship between the background knowledge of group members (as measured by the pre-test and displayed in Table 2) and improvement. Group 7 contained the only student who had completely correct answers on the pre- and post-tests, yet the others in the group did not improve. On the other hand, group 4 was effective, but the answers they gave on the pre-test showed not even a primitive "does not go" awareness of order of division. Further investigation, including detailed mapping of the prior conceptual understanding of students, is required here.

Conclusion

The task proved highly effective for reducing a persistent error, and most impressively after three weeks, for half of the class in less than half an hour. Children in effective groups were found to engage in mathematical discussion in a specific way. Additionally, although the setting was different in important respects to the settings used in other studies of peer learning, effective discussion shared many of the features found in other studies.

Both those groups which were effective and those individuals who improved interacted more than the others and they engaged at a higher level of elaboration.

The analysis of the mathematical content of the discussion showed substantial differences in engagement with the central ideas of the task between effective and ineffective groups. In summary, effective groups demonstrated different patterns of interaction from ineffective groups. They:

- Talked more, with more mathematical content (some of which was wrong).
- Explicitly discussed the central idea (order of division).
- Worked together by reading the questions on the cards 'out loud' and repeating each others' statements.
- Proposed ideas, gave explanations with evidence and refocused discussion more often.
- Responded to questions more.

Improvers made many mistakes in their mathematical talk. They were able to learn even though they made mistakes and heard others make mistakes. We hypothesise that Bell's task succeeded in creating cognitive conflict, bringing their misconceptions out into the open where the children in effective groups could grapple with them. This mechanism will be investigated in a future study. A future study will also seek a more detailed analysis of background knowledge and conceptual development of group members and improve the pre-and post-test so that growth in understanding can be measured. Tracking errors made during discussion and the resulting learning could illuminate the process of learning and reducing misconceptions through discussion substantially.

Children's discussion is widely considered to be important for learning, yet the mechanisms which make it effective are still not well understood.

However, the consistent findings on the high interaction and communication between individuals in effective groups can guide teachers towards making discussion a better learning tool. Teachers may be advised to draw their students' attention to the need for engagement with the task (e.g. by actively calculating answers to the problems given, not passively trying to match cards), for student discussion of each others' calculations and for discussion of reasons for placement of the cards during the activity. At the very least, teachers can use knowledge of the characteristics of effective groups to identify and intervene with groups which are unlikely to be learning. Schemes as proposed by Webb (1991) which train children to engage with the content of a discussion, analogous to those which train children to interact productively with each other in a group, should be investigated.

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