Aspects of knowledge development through interaction in small groups.

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Abstract

This study was designed to establish characteristics of discussion which provokes learning through cognitive conflict. Seven groups of children were video-taped during a group discussion which reduced misconceptions related to division. Transcripts were coded for mathematical and interactive aspects of the discourse. Several case studies, illustrating the range of ways in which children did and did not learn through the task are given. It is proposed that teachers who are alerted to the characteristics of effective discussion may be able to make it a better learning tool.

Introduction

This study was designed to establish characteristics of discussion which produced effective learning. Relationships between learning and social interaction are also explored. In contrast to earlier studies of characteristics of effective discussion which have been based around programs related to various co-operative learning schemes, the present study uses a task designed from a social constructionist perspective. The social interaction provides students with the opportunity to elaborate the actions of others, the opportunity to imitate others and the likelihood of conflict through which ideas can be restructured.

The extensive research on various forms of peer learning in small groups has established group discussion as an effective method of learning (Johnson, Maruyama, Johnson and Nelson, 1981; Slavin, 1989; Sharan and Shachar, 1988). Other research has focused on the internal dynamics of cooperative groups (Johnson and Johnson, 1985) and on the 'process-outcome' relationship (Webb,1985). In groups which learn effectively, members interact more with each other, interact more with the task and utilise more cognitive strategies than ineffective groups (Sharan and Shachar, 1988; Tingle and Good, 1990; Webb, 1989). Surprisingly, an analysis of the subject matter content of the talk has not been examined in these studies. Therefore this paper and also Gooding and Stacey (submitted for publication) reports on the connection between the content of group discussion and subsequent learning. In addition, examples are given of the ways in which children may learn through social interaction in general and through cognitive conflict in particular.

Whereas other studies of effective discussion have focussed on the acquisition of new knowledge, the present study focusses on the reduction of a prevalent misconception. The misconception relates to division involving numbers between nought and one. It is very well documented that many children and adults have recurring difficulties with this aspect of division. The remarkable persistence of these difficulties indicates that traditional teaching methods do not address misconceptions well. However, promising results have been reported using the "conflict teaching" approach (Bell,1986; Swan, 1983; Tirosh and Graeber, 1990) which deliberately exposes misconceptions and helps students reflect on their errors. Bell and Swan have empirically established that carefully designed group discussion tasks (including the one used in this study) are a particularly valuable component of the approach, but no analysis of the process - outcome relationship has been made. The tasks are designed to provoke socio-cognitive conflict, "a conflict, socially experienced, in which an individual's strategy is explicitly contradicted by another person's strategy". (Perret-Clermont, 1980, p31) The conflict is proposed by Perret-Clermont as the specific locus of cognitive development, and a mechanism for internalisation.

Children's discussion was videotaped during the group task. Data obtained from coded transcripts are linked to test results. Although the setting was different in important respects, effective discussion shared many of the features found in other studies. In addition, children in effective groups were found to engage in mathematical discussion in a specific way. Several case studies which illustrate different patterns of social interaction and learning are given.

Method

Choice of task and misconceptions addressed.

Concepts related to division were chosen as the mathematical content of the discussion because the difficulties are widespread across the age range from ten years to adulthood, persistent, important to address and well documented. (Bell, Greer, Grimison and Mangan, 1989; Resnick, Nesher, Leonard, Magone, Omanson and Peled, 1990; Tirosh and Graeber, 1990). Bell (1986) refined a "conflict teaching" task which has been used successfully to help children learn the following inter-related conceptions about division:

(i) that division is not commutative (i.e., that a÷b and b÷a are not interchangeable)

(ii) that division of a smaller by a larger number is possible

(iii) that division by a number between zero and one is possible and the result (the quotient) is larger than the number originally divided (the dividend).

EXAMPLE	WORDS	÷	ANS	$\overline{)\quad}$	ANS
8 apples are shared between 2 boys How many apples does each boy get?	**8 divided by 2**	8 ÷ 2	4	$2\overline{)8}$	4
2 apples are shared amor girls. How much apple does each girl get?	2 divided by 8	2 ÷ 8	1/4	$8\overline{)2}$	1/4

You have $12. Each present cost $6. How many presents can you buy?	12 divided by 6	12 ÷ 6	2	**6)12**	2
What is 6 divided by 12?	6 divided by 12	6 ÷ 12	1/2	**12)6**	1/2
4 kilometres split into 1/2 kilometre sections. How many sections are there?	4 divided by 1/2	**4÷1/2**	8	1/2)4	8
1/2 a kilometre split into 4 sections. How long is each section?	1/2 divided by 4	**1/2÷4**	1/8	4)1/2	1/8

Figure 1. The completed board showing all 36 cards correctly placed (from Bell,1986).

Cards initially in place when the board is given to students are in bold.

Confusion about the notation for division is also addressed by the task. The two commonly used notations for division work in opposite directions. Rahn's sign, ÷, puts the divisor on the right and the dividend on the left whereas the division bracket or lunar sign,) puts the divisor on the left. (Cajori, 1928) For example, 3÷6 corresponds to 6)3 and not to 3)6. In English, the confusion is also reflected in the associated verbalisations so that 3÷6, for example can be read with the numbers in the same order as "3 divided by 6" or with the numbers in the opposite order as "6 (divided) into 3".

In the task, children have to place 36 cards in appropriate places on a grid drawn on a rectangular board. Figure 1 shows the correctly completed placement of cards on the board. Six cards are associated with each division question: a word problem, a verbal statement of the calculation, the division question written using Rahn's sign and the lunar sign and two answer cards. The division questions come in pairs: the first requiring a larger number to be divided by a smaller number (e.g. 6)12 and 12÷6) and the second requiring the much harder reverse situation with the same numbers (e.g. 12)6 and 6÷12). Henceforth, the former divisions will be called larger/smaller divisions and the latter divisions will be called smaller/larger divisions.

Procedure

Single-sex groups of four children from a class of 28 children (average age 10.5 years) were videotaped working on the activity. Children were presented with the board, which had headings and a selection of the cards (those in boldface type in Figure 1) already in place. They were instructed to place the other cards in the appropriate places on the board and talk about what they were doing as much as possible. When the board was complete, they were asked to compare their board with one which was nearly correct and said to have been organised by another school. This gave them an opportunity to correct their own board and discuss any remaining misconceptions.

A pre-test was administered one week before the activity and the post-test was administered three weeks after it. There were nine questions in both tests of which four smaller/larger divisions were the target questions: 6)3 and 4 ÷16 on the pre-test and 6)3 and 5÷25 on the post-test. The classroom teacher gave the answers to the pre-test immediately afterwards but did not teach this topic.

The videotapes, including all of the discourse and all gestures which indicated the order of division operations, were transcribed. Two coding schemes were developed. A modified version of Sharan and Shachar's scheme which focused on aspects of the interaction was used to code the discourse. The second scheme coded the mathematical content of children's discourse in six categories which are given with illustrations in Figure 2.

Individual students who were "improvers" and "non-improvers" were defined by gains on the target questions from pre-test to post-test, as were effective and ineffective groups. Summary data from the coded transcripts were related to the test results. Complete copies of the transcripts are held by the authors.

Results

Children talked and placed and pointed at the cards to reach consensus, taking from 10 to 25 minutes to complete the task. When they announced they were finished, three of the seven groups had correctly placed the cards on the board. The results on the target test questions and the relationships between these and the interactive and mathematical behaviour of the groups are given in detail by Gooding and Stacey (submitted for publication). A summary is presented here. Learning was generally effective. Half the class improved on the difficult smaller/larger division questions. These children will be designated "improvers". The other children are designated as "non-improvers", except for the child who had a perfect score on both tests.

Table 1. Improvement on smaller/larger divisions and amount of discussion.

Group	Net* Gain	No. of students correct on pretest		Effective or not	No. of improvers	No. of turns talking
		6)3	4 ÷ 16			
1. boys	+4	0	0	Eff	2	399
2. boys	+4	1	0	Eff	3	362
3. girls	+5	1	0	Eff	3	609
4. girls	+3	0	0	Eff	3	407
5. girls	+1	0	0	Not	1	297
6. boys	+3	1	0	Eff	2	820
7. boys	0	1	1	Not	0	174

* Net gain from pre-test to post-test on the two smaller/larger divisions totalled over all group members.

Five groups were considered to be effective, and two groups (one of boys, one of girls) ineffective. In each of the effective groups, there were at least two improvers. Conversely, in every group, there was at least one non-improver. The details are shown in Table 1. Table 1 also gives a crude measure of the amount of talk was estimated from the number of turns students took on the transcript, as defined by Sharan and Shachar (1988). The two ineffective groups had students taking the fewest number of turns talking.

The coding of the discourse showed that children in the ineffective groups interacted less than those in the effective groups. More of their talk was in the "thinking aloud" category and less in categories such as responding. Members of effective groups gave more explanations with evidence, repeated each other's statements more frequently, and responded and explained more during the task. Children in ineffective groups asked questions of others relatively more often than the others. However, they received relatively fewer responses. A similar observation has been made by Webb (1989) whose proposition that high-level elaboration of help given is related to achievement was also confirmed.

Correct smaller/larger statement:

Hugh: *1/2 kilometre split into 4...... 1/8 of a kilometre is how long is each section.*

Incorrect smaller/larger statement:

Julian: *How many 12s are in 6?*
Anthony: *2*

Correct larger/smaller statement:

Chris: *4 how many 1/2s means*
Anthony: *8*

Identifying non-commutativity:

Anthony: *8 divided by 2 and 2 divided by 8. You've got two different things, two different things.*

Correct discussion of the order of division operation:

Hugh: *That's 12 by 6* . [Pointing to 6)12 right to left]

Incorrect discussion of the order of division operation:

Anthony: *8 how many 2s* [Pointing to 8)2 left to right]

Figure 2. Examples of each category of mathematical discussion.

Despite the differences between the setting and task investigated in this paper and the settings and tasks of previous studies of characteristics of effective discussion, the broad patterns of interaction which have been associated with higher achievement of groups were also found in this new setting. The same broad pattern of interaction was observed at the individual level, contrasting improvers and non-improvers. A full discussion is given by Gooding and Stacey (submitted for publication).

Mathematical content of the discussion

Analysis of the mathematical characteristics of the discussion and gesturing (indicating order of division) in several categories indicated striking differences between effective and ineffective groups. The categories are illustrated in Figure 2. As is shown in Gooding and Stacey (submitted for publicaton), there was hardly any explicit mathematical discussion in the ineffective groups. The number of instances in each category of mathematical content (see Table 2) for the two ineffective groups was less than the corresponding number of instances for the effective groups. The Mann-Whitney test showed this to be significant at the 5% level. Parallel differences were found between improvers and non-improvers.

Table 2. Median number of statements in specific categories of mathematical talk

	Correct smaller/ larger	Incorrect smaller/ larger	Correc larger/ smaller	Identifying non-commut ativity	Correct discussion of order	Incorrect discussion of order
Groups						
Medians for effective groups	7.0	17.0	8.0	10.0	10.0	6.0
Medians for ineffective groups	0.5	4.0	0.5	0.5	0.0	0.0
Significance of difference	*	*	*	*	*	n.s.

* significant at the 5% level, n.s. not significant

Neither of the ineffective groups specifically discussed the order of division operation for either Rahn's sign or the lunar sign, whereas the effective groups discussed this explicitly 8 or more times each. This is an important difference. The ineffective groups each made only 5 calculations of any sort. Each of the effective groups made at least 27 calculations out loud. It is important to point out that these differences were not obvious until a close analysis of the discourse was made. We do not believe that a teacher circulating around the classroom whilst students were working, would have been able to spot the differences in engagement - all groups appeared to work enthusiastically on the task. Members of all groups made mistakes calculating smaller/larger divisions. Overall, both improvers and non-improvers made more incorrect than correct calculations when saying smaller/larger divisions "out loud". Perhaps surprisingly, the improvers made more mistakes with the smaller/larger divisions than did the non-improvers. However, a smaller percentage of their calculations were incorrect. These students were able to learn even though they made mistakes and heard others make mistakes.

In summary, Bell's task succeeded in creating cognitive conflict, bringing their misconceptions out into the open where the children in effective groups could grapple with them. In Figure 3, the differences between the effective and ineffective groups are summarised.

Figure 3. Summary of differences between effective and ineffective groups.

Effective groups:

- Talked more
- Frequently stated divisions, both correctly and incorrectly.
- Explicitly discussed order of operation and non-commutativity
- Worked together by reading the questions on the cards "out loud' and repeating each others' statements
- Proposed ideas, gave explanations with evidence and refocused discussion more often
- Responded to questions more

Ineffective groups:

- Talked less and their talk contained less mathematical content
- Stated very few divisions, correct or incorrect.
- Made absolutely no explicit discussion of order of operation.
- Did not read the questions on the cards "out loud" at all and seldom repeated each others' statements
- Seldom proposed ideas, gave fewer explanations with evidence, and refocused the discussion less often
- Spent more time on unfocused interaction, just thinking out loud.

Individual Patterns of Learning

Although the previous sections show that at the macroscopic level there are important commonalities in the processes that promote learning, at the microscopic level learning is much more variable. There is great variety in the way in which individuals interact and how they learn. In this section, some of this variety will be sketched. The case studies below illustrate some of reasons behind the group dynamics and how these contributed to instances of learning.

Before exploring the case studies however, it is important to acknowledge that, except in fortuitous circumstances, our methodology biasses the instances of learning which we capture to those with associated utterances. Learning, even in a group discussion task, is sometimes not associated with talk at all. For example, in the ineffective group of girls, no correct mathematical talk occurred, yet one girl, Anna, did improve. The videotape indicates that she learnt by looking at the correct answers on the board said to be from the other school. After carefully comparing her group's placement with the introduced board, she changed her cards to agree except with the deliberate error on the other school's board.

Emma - patiently taught, by discussion of her errors.

Emma, in group 3, had neither of the pre-test questions right but got both of the post-test questions right. Her statements near the beginning of the discussion revealed that she did not know that the two division notations operated in opposite orders, although she did realise that division was not commutative and that a small number could be divided by a larger number.

The transcript clearly reveals that Emma was taught patiently by two of the three group members, who commented on her incorrect placement of cards six times. This contrasts with other groups in which members commented on incorrect mathematical statements, but not on card placement. Emma practised correct order at the end of the activity, by verbalising each card when the group was reviewing their work. The transcript below illustrates one of the lengthy teaching segments in which the group engaged.

113. Emma *Is it 4 into 1/2 or 1/2 into 4?*
114. Vicki *1/2 km is split into 4 sections*
115. Emma *That's the other way round*
116. Vicki *Is it?*
117. Emma *And then that one goes there. I was looking the other way round*
118. Amy *Yeah*
119. Emma *It's that way, that way*
120. Marnelle *No, no,see . . .because it's how many 1/2s in 4*
(correcting Emma's reading of 4 ÷ 1/2)
121. Vicki *8*
122. Marnelle *So you say 1/2 into 4*
(Pointing correctly to 4 and 1/2 on the card)
123. Emma *Mmm, OK, yeah, its that.*

The example of Emma shows a real strength of group discussion. It is hard to imagine a class teacher able to explain almost the same point to one child ten times in less than twenty minutes.

Hugh and Julian - resolving conflict

Not all succcessful learners were taught as patiently as was Emma. For example Hugh, from Group 6 which also contained Julian, Antony and Chris, learned much more quickly and also played a more active role in the group. On the pre-test and also at the beginning of the activity, Hugh could carry out large/small divisions correctly with the lunar sign, did not recognise that order matters and was confused about the order for Rahn's sign. Early in the discussion, he questioned the notion that some answers could be fractional. These confusions were eliminated after three interactions. On two occasions Hugh made errors with order and was corrected by Julian. The extract below is part of a long interchange on the third occasion with both Antony and Hugh where Julian explicitly drew attention to the difference between the two division signs. Julian treats the cards in order, right to left across the board and points to each number when he says it.

166. Julian *How many 6s in 12?* (lunar sign card) *How many 6s in 12?* (Rahn's sign card) *12 divided by 6. How many 6s in 12?* (rephrasing words card) *How many 6s in 12?* (pointing to numbers from example card)
167. Hugh *That how many 12s in 6* (12 ÷ 6 - wrong)
168. Julian *No. Its how many 6s in 12* (pointing again) *How many 6s in 12? How many 6s in 12?* (moving along to word statement card)
169. Hugh *Yeah, 12 how many 6s.*

Hugh practised this new knowledge as he placed later cards (correctly), seeking confirmation from Julian. Note how Hugh's final summary in statement 169 actually uses words not used by Julian. He is not imitating, but connecting the new information with previous knowledge. Despite his clear explanation, Julian himself had difficulty with Rahn's sign (but not with the lunar sign). He had written 4 ÷16 = 4 on the pretest and during the discussion, he used Rahn's sign incorrectly two out of five times. On these two occasions he was corrected by Hugh and Antony. He was correct on the pre-test. Julian therefore was both a teacher and a learner.

Antony - concerns not addressed

Antony had written on the pretest 4 ÷16 = 4 and 6)3 = 18. During the discussion, he read out the order of two divisions using Rahn's sign correctly but made errors of order on all of the three lunar sign divisions which he calculated out loud. Twice he was corrected, but he did not practise what he had been shown. At the very end of the activity, unobserved by his colleagues, he interchanged two of the lunar brackets cards which had been correctly placed on the final board. Antony did not learn about the order of the lunar sign from the discussion.

On the post-test however, after three weeks, Antony was able to distinguish both of the smaller/larger divisions and answered them in the zero + remainder form. Antony in fact suggested this form of the answer three times during the discussion, starting right at the beginning of the discussion by objecting to Julian's statement that

"6 divided by 12 is 1/2", by saying it should be "zero, remainder 6". On this occasion the comment was ignored by the others. On the other two occasions, Julian and Hugh supplied the correct fractional answer, but they did not discuss Antony's amswer at all. Antony subsequently gave correct fractional answers seven times during the discussion. Despite this, on the post-test he reverted to the zero + remainder form. We conjecture that this may have been because the question of the validity of the zero + remainder form was never addressed in the discussion.

Chris - insufficient background knowledge.

Chris, the fourth member of this group has played no part, other than as an observer in the activity described above. He played a minor role in the discussion, taking only 13% of the turns talking in the group. He made many mistakes, was constantly corrected by the others, and showed no improvement from the pre-test to the post-test, although during the discussion he did begin to show an awareness of order of Rahn's sign only. We conjecture that Chris was unable to benefit from the discussion because he did not have a sufficiently firm knowledge of fractions. For example, on one occasion he misread 1/2 as 12, and he could not mentally calculate items such as "how many 1/2 km sections in 4 km". Therefore his only practicable course was to work entirely in whole numbers. Chris did not get the help that he needed from this activity - the new knowledge was not in his zone of proximal development (Vygotsky, 1978). The example of Chris illustrates one of the reasons why the non-improvers were found to contribute so little to the mathematical aspects of the discussion. Children who are unable to cope with the task are less able to exchange mathematical ideas and explain them clearly. This results in a greater percentage of talk of non-improvers in the unfocussed "thinking aloud" category.

Order matters

Julian Chris *

Antony Hugh *

Some answers fractional

Julian Chris

\# Antony Hugh *

Order for lunar sign

Julian Chris

* Antony Hugh

Order for Rahn's sign

* Julian Chris

Antony Hugh *

A -----> B indicates an explanation given by A to B

* indicates that the child learnt from the help given

\# indicates that child did not retain understanding demonstrated in discussion

Figure 4. How information was shared between Julian, Hugh, Antony and Chris.

Discussion

Background knowledge and learning

All effective groups contained individual members who did not improve, despite the mathematical nature of the effective groups' discussion. Non-improvers did not learn, even though they heard and often joined the mathematical discussion (although to a lesser extent than improvers). It is likely that a threshold of background knowledge was required to gain long-term benefit from participation in the task. An individual example of the importance of background knowledge is afforded by Chris, whose understanding was sketched above: smaller/larger division seemed not to be within his zone of proximal development. As Perret-Clermont (1980) has noted, sensitivity to conflict has as its pre-requisite the ability to understand the question at issue in an exchange.

Groups as well as individuals require adequate background knowledge to benefit from the task. For example in Group 6 where all but Chris improved, although no individual could do the task alone, all the required knowledge was present initially in the group. This is shown in Figure 4, which also traces how the information is shared. Both Antony and Julian initially knew that order of division matters, and could do some smaller/larger questions. Hugh and Julian knew the order for the lunar sign and Antony knew the order for Rahn's sign. Julian knew that some answers would be fractional. Together, they had all the required knowledge and were able to share it, except with Chris.

On the other hand, group 4 was effective, but the answers they gave on the pre-test showed no awareness that order of division was important at all. However, the early discussion on the task did indicate that the pre-test was not sufficiently sensitive to capture all the background knowledge students had. In our next study, we plan to carry out more detailed mapping of the prior conceptual understanding of students, in order to establish whether learning occurs only by sharing prior knowledge and clarifying it, or whether new information can be injected from the board and task structure itself.

One group demonstrated dramatically that having all the required background knowledge is not sufficient for the group to learn. The only group in which no-one made gains contained the only member of the class who could calculate smaller/larger items correctly from the beginning. He did not explain to others and they placed cards with the least discussion.

Task structure

Webb (1989) proposed that task structure, which was not investigated at all in the 19 studies she reviewed, has a powerful effect on the interaction in small group work. In our opinion, this task is well structured and does promote effective learning. However, it is important to note that its effectiveness relies on students spontaneously deciding to practise target skills (such as identifying the order of division) or spontaneously deciding to explore ideas (such as the zero + remainder form) which arise. Some students will be more likely to engage in this way than others. Probably individual learning styles, confidence and a feeling that the material is not out of reach are key factors. This task covers the subject matter of the misconception very well. Students seem to enjoy the board game nature of the task, but ultimately attaining a sufficient level of engagement in the task will be socially determined.

Conclusion

The conflict teaching task proved highly effective for reducing a persistent error, and most impressively after three weeks, for half of the children in less than half an hour. The overall pattern of discourse encouraged by this task appeared to be somewhat different to discourse in other co-operative tasks that have been reported in the literature, especially in that all students and all groups spent a large proportion of the talk on unfocused "thinking aloud". This probably reflects the high cognitive demand of the task and its relatively unstructured nature. This task is much less structured than other co-operative teaching techniques which have been studied. This feature may be one of the characteristics of the task which make it suitable for reducing misconceptions, since both Slavin (1980) and Ross and Raphael (1990) have noted that less structured co-operative techniques are especially suitable for higher level cognitive learning. With this exception, the pattern of differences between effective and ineffective groups was similar to the differences reported in the literature. Both those groups which were effective and those individuals who improved interacted more than the others and they explained more and gave more extended responses.

The analysis of the mathematical content of the discussion proved to be particularly revealing. Most strikingly, the children who did not learn well did very few calculations of any sort and never discussed the importance of order of operation. The children who did learn calculated out loud, and repeatedly discussed the order of the division operations and non-commutativity. They appeared to have sufficient background knowledge to engage with the mathematics and to articulate their thoughts on it. The task was within their zones of proximal development. Nevertheless the presence of background knowledge amongst the group members is not a sufficient condition for learning. The children who were shown or told the answers by the student who had a perfect score on both tests did not learn.

Much of the mathematical talk in both effective and ineffective groups was wrong. Despite this, or perhaps because of it, many children learned and retained what they learned for three weeks. In responsive, interactive groups, some of the errors generated cognitive conflict, which in turn generated discussion and thinking, which in turn generated long-term learning. A detailed analysis of background knowledge and conceptual development of group members, errors made during discussion and the resulting conflict and learning could illuminate the process of classroom teaching through cognitive conflict substantially.

Learning is taking place in the activity through both imitation and conflict causing restructuring of ideas. Our next study will seek to examine in more detail the role played by cognitive conflict as a mechanism of learning. Other examples on the transcripts indicate that children generally do not seem to know they are wrong, or give evidence of conflict, when they use a sign in the wrong order. The cognitive conflict seems first to arise in the listener (or the student watching another misplacing a card), not the speaker. The confusion is seldom recognised initially by the speaker.

A social constructionist perspective on learning has been shown to have clear application to reducing the incidence of misconceptions. Although the mechanisms by which children learn in groups are still not well understood, it seems that sufficient information is now available to help teachers promote quality learning through group work.

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