Technology Enriched Algebra in Year 9[1]

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Introduction

Computer spreadsheet and graphing applications are useful tools for investigating mathematical procedures and problem situations. They can quickly provide graphic and numeric representations of the symbolic form of a mathematical model which can assist the problem solving process but they do not produce, by themselves, the solution to a problem. The creation of an adequate model and the correct interpretation of its various representations are also needed if a solution to a problem is to be found.

Consider this problem, which is an adaptation of one suggested by Bolt (1986, p. 58).

> Four large towns lie at the vertices of a square of side 20 km. They are to be connected to each other by a road network of minimum length. At first it was thought that the network shown in Figure 1 gave the minimum length. However, this is not true. Find a road network having a smaller length.

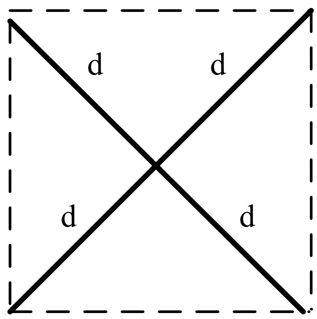

Figure 1.
Road Network with Length 4d

In seeking to find a shorter network, a more general road pattern which includes that in Figure 1 as a special case could be investigated and systematically varied to check to see if a smaller road length could be found. One possible road pattern is that shown in Figure 2.

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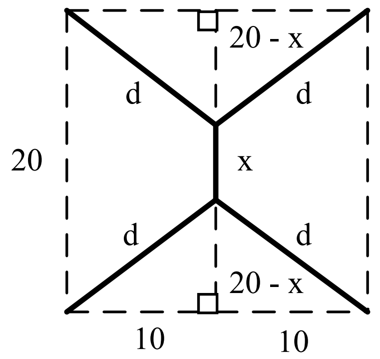

Figure 2.
Road Network

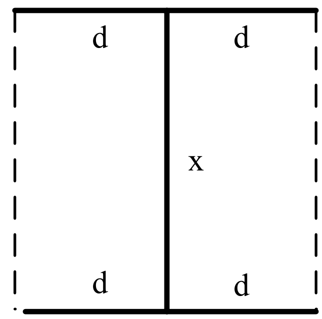

Figure 3.
Road Network when x = 20

Networks of this shape have a total road distance given by the function T with rule $T(x) = 2\sqrt{x^2 - 40x + 800} + x$, $0 \le x \le 20$. When x = 20, the network reduces to that shown in Figure 3 and T(20) = 60. When x = 0, the network becomes that shown in Figure 1 and $T(0) = 40\sqrt{2} \approx 56.5685$. This model will provide a shorter network if a value of x between 0 and 20 can be found for which the road length is smaller that $40\sqrt{2}$. However, if this model does not produce a value for x for which $T(x) < 40\sqrt{2}$ it does not mean that the problem cannot be solved, since a different road pattern may produce a solution.

Using a computer graphing application the graph of T can be quickly drawn to give the curve shown in Figure 4. From the graph it appears that there is a range of values for x which produce values for T smaller than $40\sqrt{2}$ and that the value of x which gives this minimum is somewhere between 8 and 9. The coordinates of the minimum point can be read directly from the graph by use of options available in the graphing application.

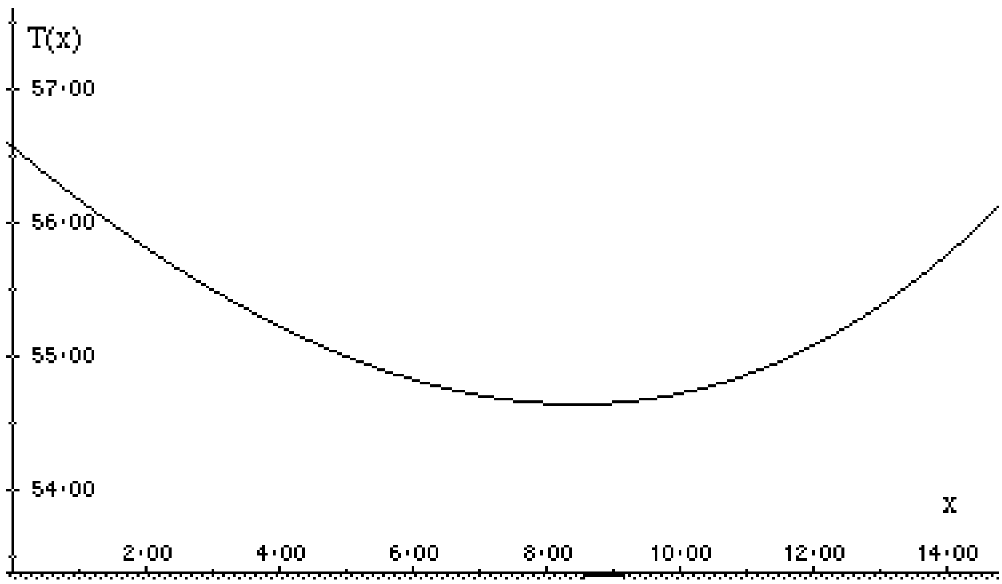

Figure 4.
Graph of T

As an alternative, the spreadsheet shown in Figure 5 can be used to find, for 8 < x < 9, the minimum value of T and the value of x for which this occurs. This is done by systematically using information in a spreadsheet screen display to create a new display by entering a new initial x value for the table (cell A4), a new "step x" number (cell B1) or both. If the minimum in the table occurs in the last row of the table then, without changing the step value, the x value which gives this table minimum must be entered as the initial x value for the table. If the minimum occurs in the first row of the table, the initial x value remains and only the step number is changed from k to $\frac{k}{10}$. If the minimum occurs within the table, the x value before the x value which gives this minimum is entered as the new initial x value and the step number is changed from k to $\frac{k}{10}$. Starting with 8 as the initial x value and 1 as the step value, the spreadsheet screen display in Figure 6 was developed in this manner to find, correct to six decimal places, the value of x between 8 and 9 for which the function T is a minimum.

	A	B	C	D	E
1	Step x =	.0000001			
2				Minimum	=Min(B4:B13)
3	x	T(x)		YES = 1 , NO = 0	
4	8.452994	=2*Sqrt(A4^2-40*A4+800)+A4		=If(B4=$E$2,1,0)	
5	=A4+$B$1	=2*Sqrt(A5^2-40*A5+800)+A5		=If(B5=$E$2,1,0)	
6	=A5+$B$1	=2*Sqrt(A6^2-40*A6+800)+A6		=If(B6=$E$2,1,0)	
7	=A6+$B$1	=2*Sqrt(A7^2-40*A7+800)+A7		=If(B7=$E$2,1,0)	
8	=A7+$B$1	=2*Sqrt(A8^2-40*A8+800)+A8		=If(B8=$E$2,1,0)	
9	=A8+$B$1	=2*Sqrt(A9^2-40*A9+800)+A9		=If(B9=$E$2,1,0)	
10	=A9+$B$1	=2*Sqrt(A10^2-40*A10+800)+A10		=If(B10=$E$2,1,0)	
11	=A10+$B$1	=2*Sqrt(A11^2-40*A11+800)+A11		=If(B11=$E$2,1,0)	
12	=A11+$B$1	=2*Sqrt(A12^2-40*A12+800)+A12		=If(B12=$E$2,1,0)	
13	=A12+$B$1	=2*Sqrt(A13^2-40*A13+800)+A13		=If(B13=$E$2,1,0)	

Figure 5.
Spreadsheet to Find x for Minimum T(x)

Step x =	0.0000001			
			Minimum	54.6410161513775459
x	T(x)		YES = 1 , NO = 0	
8.452994	54.6410161513775582		0	
8.4529941	54.6410161513775545		0	
8.4529942	54.6410161513775515		0	
8.4529943	54.6410161513775491		0	
8.4529944	54.6410161513775474		0	
8.4529945	54.6410161513775463		0	
8.4529946	54.6410161513775459		1	
8.4529947	54.6410161513775461		0	
8.4529948	54.641016151377547		0	
8.4529949	54.6410161513775485		0	

Figure 6.
Spreadsheet displaying x for minimum T(x) when 8 < x < 9

If students are to make use of spreadsheet and graphing applications in problem investigations in the manner illustrated above, they need to view these programs as essential, readily available resources which will help them to learn and apply mathematical ideas. In order for this to occur, they need to acquire adequate skill in their use and an awareness of the nature, strengths and limitations of the representations they can provide. This should be done in the early years of secondary school when students are constructing and developing their conceptual knowledge and skills associated with algebra, graphing and functions. If this is done properly, the technology will not only assist students to learn mathematical concepts, skills and procedures which are essential for problem solving but will provide insights as to the nature of the contributions the technology can make to the problem solving process. As Ruthven (1992) suggests, the technology may then become a "cognitive tool" for thinking and learning.

A Use of Spreadsheet and Graphing Applications in Year 9 Algebra

The *Technology Enriched Algebra Project* (TEA) at the University of Melbourne set out to develop, trial and evaluate instructional activities and material using spreadsheet and graphing applications which might help students to learn ideas related to linear equation solving at year 9, and to research the impact of the classroom use of the computer applications and associated instructional materials upon the development of basic algebraic concepts and procedures.

The instructional materials developed were trialled during a three week period in April - May 1992 at Lalor North Secondary College in six year 9 classes taught by four teachers. Three classes were taught with materials based upon the use of Macintosh spreadsheet (MSWorks) and graphing (ANUGraph) applications. The other three classes were taught with similar instructional materials using both teacher provided tables and graphs and student constructed tables and graphs. The remainder of this paper will briefly describe the TEA computer materials and some results relating to their use.

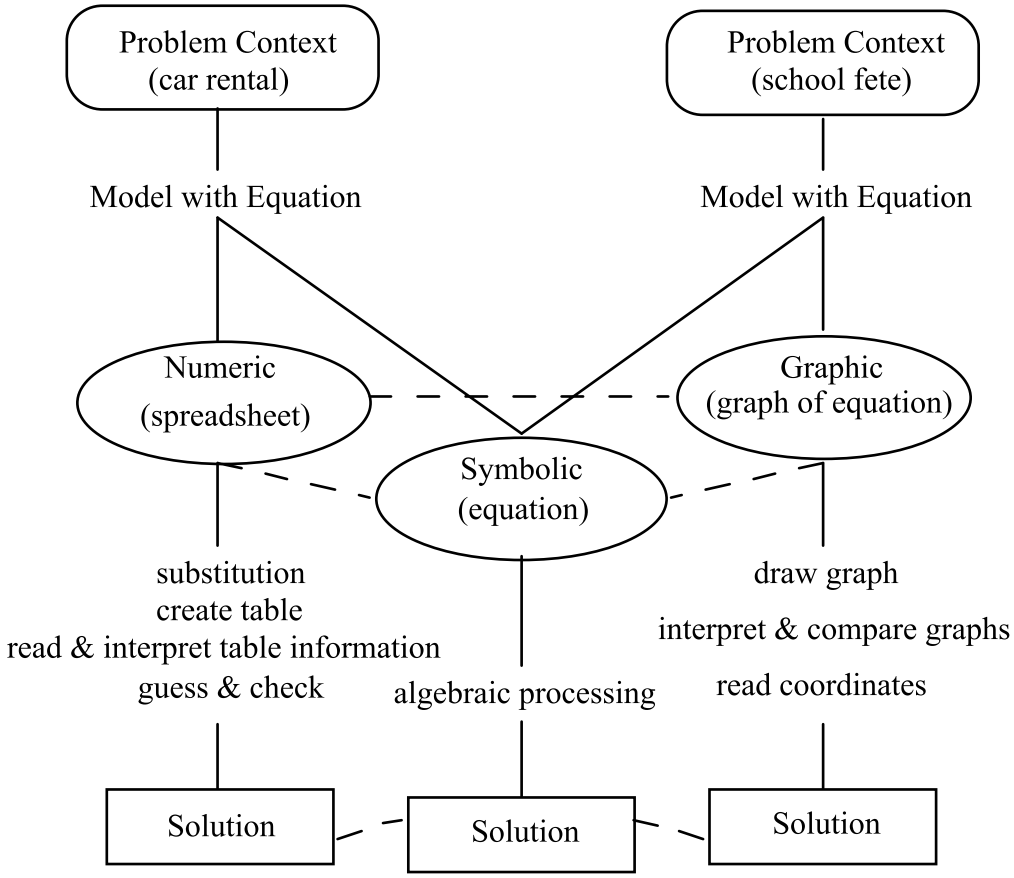

Figure 7.
Three methods of equation solving

Aims
The main aims in the design and development of the TEA Project Year 9 computer materials were to provide opportunities for students to: (a) learn to use spreadsheet and graphic applications to find the solution of a linear equation; (b) compare and link different representations of an equation; (c) interpret information provided by each representation; (d) use their understandings of the different representations to explore relationships between algebraic expressions.

In addition, it was felt that the use of the spreadsheet and graphing applications within the context of the instructional program would contribute to the development of student understandings related to each item listed below.

1. A letter is used to represent a variable number.
2. An expression is a variable number, dependent upon the value of the variable in which it is defined.
3. An equation is a statement that the numbers named by two expressions are sometimes (but not necessarily always) equal.
4. An identity is a statement that the numbers named by two expressions are always equal.
5. To solve an equation means to find the number(s) which make the two expressions which define the equation give the same number.

6. The solution(s) to an equation can be found by a guess and check approach and this process can be aided and refined by using relevant information in tables and spreadsheets.
7. The solution(s) to an equation can be found by a graphic approach by graphing the functions corresponding to each side of the equation and reading the coordinates of the points of intersection of the graphs.

Approaches to Equation Solving

As well as the usual algebraic manipulation procedures, computer generated numeric and graphic representations of an equation can be used to solve an equation. The numeric and graphic equation solving methods used by Asp (1991) in a bridging mathematics subject at the University of Melbourne formed the basis of the computer approaches to equation solving. The TEA Project materials make use of all three approaches (symbolic, numeric, graphic) to equation solving so that students may learn to link relevant ideas provided by the different representations of an equation and learn to use these representations to derive appropriate information when analysing the properties of the equation. Figure 7 from Asp, Dowsey & Stacey (in press), summarises the approach to equation solving found in the TEA Project materials .

Numeric equation solving. In numeric equation solving the student uses a given spreadsheet to simultaneously create tables for two functions by changing the initial x value for the tables, the step number by which each successive x value for the tables is generated or both. To solve an equation, a primitive guess and check strategy can be used or a more refined guess and check approach can be developed by the student in which successive spreadsheet displays are created which progressively give closer approximations to the solution. The latter requires the judicious selection of alterations to the initial x value and step number based upon careful examination of the tables shown in the current spreadsheet display. Figure 8 shows a typical spreadsheet display giving the solution to the equation F(x) = G(x) or
4 + 0.25x = 8 + 0.15x.

Step	x	F(x) = 4 + 0.25x	G(x) = 8 + 0.15x	F(x) - G(x)
10	0	4	8	-4
	10	6.5	9.5	-3
	20	9	11	-2
	30	11.5	12.5	-1
	40	14	14	0
	50	16.5	15.5	1
	60	19	17	2
	70	21.5	18.5	3
	80	24	20	4
	90	26.5	21.5	5
	100	29	23	6

Figure 8.
Example of numeric equation solving

The TEA Project materials did not require students to design and originally set up a spreadsheet to solve an equation. The appropriate spreadsheet for each equation was already located in the student's folder on the computer's hard disk and a spreadsheet was opened from the folder when needed.

Some numeric equation solving activities were designed to encourage students to perform simple algebraic manipulations in order to get the equation in a form which would allow for the full and immediate use of the information provided by the spreadsheet. For example, in order to use the spreadsheet in Figure 8 to solve the equation F(x) + 3 = 12 + G(x) it would be best to first rewrite the equation as F(x) - G(x) = 9 so that the values for F(x) - G(x) given by the spreadsheet can be efficiently used in seeking a solution to the equation.

Graphic Equation Solving. In graphic equation solving the student uses the computer graphing application to draw the graphs for the two functions defined by the right hand side and the left hand of the equation and finds the solution to the equation by reading the coordinates of the point of intersection. Figure 9 shows the computer graphic solution to the equation 4 + 0.25x = 8 + 0.15x.

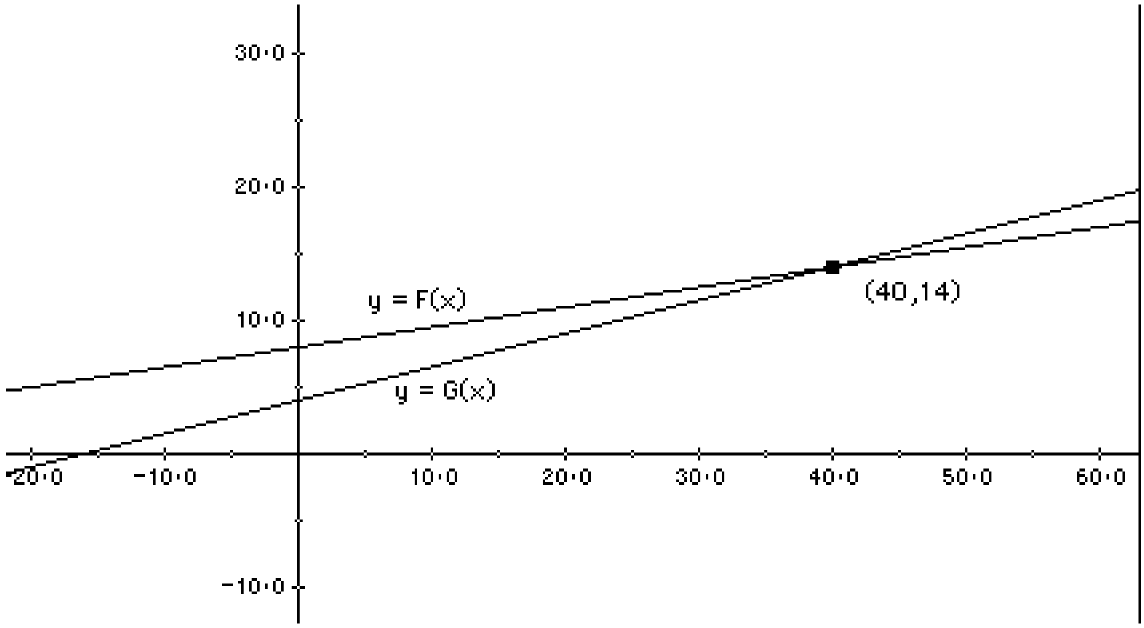

Figure 9.
Example of Graphic Equation Solving

Finding Rules for Functions
As well as solving equations, the TEA Project materials ask students to find rules which were used to generate a table of values. The rules are hidden in the spreadsheet and the computer uses the rules to generate a table of values for a range of specified x values. Students are only allowed to change the initial table value and the step number to create a new table of values. By creating new tables in this way and using the information that a table provides, the rule for the function can be discovered.

Sometimes, Always or Never
The TEA Project materials also provide opportunities for students to use spreadsheet and graphing applications to compare two expressions in order to determine if they are sometimes, always or never equal. The spreadsheet is used to create different tables of values for the expressions in order to do this. By comparing the numbers named by the two expressions over a wide range of values for the variable in which the expressions are defined, the students can decide if the expressions are sometimes, always or never equal. If they are thought to be always equal, an algebraic explanation was sought. Also, by graphing the

function which gives the number named by each expression on the same set of axes, the relationship between the graphs produced will determine if the expressions are sometimes (graphs intersect), always (one graph) or never equal (no intersections). For the expressions which are sometimes equal, the values for which they are equal must be found and for expressions that are never equal one or both have to be altered in order to make them always equal.

Results from Trial Use of Year 9 TEA Project Materials

The Year 9 TEA Project instructional materials were first used by teachers and students at Lalor North Secondary College. The school had recently acquired a room equipped with sufficient Macintosh computers so that it could be used by class with two students to each computer. The students and some of the teachers themselves had little prior experience in the use of the spreadsheet and graphing applications but both groups were eventually able to acquire adequate skill in their use to approach the tasks in the lessons with confidence. Perhaps if students had initially been more familiar with the skills associated with the use the computer and the applications more of their attention could have been given to the ideas being developed in the early lessons.

Students did learn to use a spreadsheet to solve an equation by means of a guess and check approach. This was accompanied by a good understanding of what it means to solve an equation, although few students had good manipulative skills. However, their guess and check approach rarely made full use of the information presented in the spreadsheet display to select new step values and initial x values with which to create a new spreadsheet display. Some students seemed only to use the top line of the spreadsheet, not appreciating the range of information available at each time. It appeared that students could not discover without direct instruction the coordinated use of the step and initial x numbers to develop a systematic guess and check approach which found the solution through a succession of spreadsheets with each new display giving a closer approximation to the solution. More direct teaching on these strategies is indicated.

Students were able to quickly and successfully learn to use ANUGraph to draw graphs and solve equations. This may have been due to the fact that the lessons using ANUGraph followed those making use of spreadsheets so that students were very familiar with the skills of using the computer. Alternatively, it may be that the visual approach to solving equations is more easily grasped by students and should be used before the use of a spreadsheet approach.

The nature of the normal classroom dynamics changes when students are working at computers. Student attention is focused upon the computer screen display and there is a reluctance on the part of both the teacher and the student to break from the involvement with the computer to engage in the usual classroom reflective activities which would highlight and emphasise key ideas and procedures being developed. It is important for students to be asked to think about and discuss aspects of what they are doing with the computers and not be allowed to simply "do the work" on the computer.

Technology is changing fast and new tools always seem to be just around the corner. Whilst acknowledging this, we believe that it is now time for schools to begin to develop comprehensive plans for incorporating computer tools into their mathematics curricula. Most schools now have the hardware (although perhaps not enough of it) to run central mathematics applications such as spreadsheets and computer graphing packages. Many common spreadsheets are suitable for school use and with ANUGraph for the Macintosh and CAPGRAPH for IBMs we have access to excellent and inexpensive graphing packages. From Year 9, if not earlier, students should begin to learn how to use these tools, both whilst developing new concepts as in the TEA project and in solving problems. When they know the software and are confident in operating the school machines, they will be able to use them throughout their mathematics career, particularly in problem solving and project work. When students get constant exposure to these tools, the real benefit of the technology will be realised.

References

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