Teaching Mathematics with Technology — Computer Spreadsheet and Graphing Applications

Gary Asp, John Dowsey and Kaye Stacey
The University of Melbourne

Introduction

Computer spreadsheet and graphing applications are two essential technology tools that *every* year 7 – 10 student should use in their study of mathematics. The reason is simple: properly applied to appropriate learning situations, these tools can help students to learn mathematics and learn it better. In addition, they are readily available in schools and, in mainstream mathematical content contexts, it is easy to learn how to use them. With the use of these tools there is potential for students to:

- learn mathematical topics at a deeper and more meaningful level
- interact with important mathematical ideas earlier than previously possible
- link different modes of representation of mathematical ideas
- gain valuable approaches for problem solving and investigations

In the Technology Enriched Algebra Project at the University of Melbourne (Asp, Dowsey, Hutton, McLennan & Stacey, 1992a,b), we are studying how these tools can be used in schools so that these potential advantages are achieved. We are collecting data on how students' understanding of algebra is changed and what teaching methods are successful (Asp, Dowsey & Stacey, 1992, 1993).

In this paper, we give three examples of different ways in which teachers might use spreadsheets and graphing applications. The first is as an initial introduction to spreadsheets which could also help to develop students' ideas of variables. The second uses spreadsheets as a problem solving tool and also shows a dynamic linking between formulas, tables and graphs. The third example illustrates an open-ended investigation which can be undertaken at a number of levels. These are illustrative of the types of activities that are being investigated in our research.

Spreadsheets

Example 1: Introducing Spreadsheets and Variables

Spreadsheets can be used to quickly find the number produced from a specific starting number by a sequence of operations performed in order on the successive results of each operation, a procedure which will be familiar to teachers who have used a 'backtracking approach' to the solution of equations (Giles, 1979). This provides an effective and stimulating way to introduce students to the notion of an algebraic expression as a variable number dependent upon an initial starting number and is also a useful vehicle for learning to create simple rules in a spreadsheet.

	A	B
1		
2	Choose a Number	
3	Add four	
4	Multiply by three	
5	Divide by two	
6	Subtract three	
7		

Figure 1a

	A	B
1		
2	Choose a Number	
3	Add four	=B2+4
4	Multiply by three	=3*B3
5	Divide by two	=B4/2
6	Subtract three	=B5-3
7		

Figure 1b

In this introductory activity, the student opens a teacher prepared spreadsheet file which has the "Show Formulas" option selected and contains a list of operations in column A (see *Figure 1a*). Next the student clicks on cell B3 and types ***= B2 + 4***, thus making the number in B3 equal to the number in B2 plus 4, and proceeds to enter the other B cell rules in order to perform the operation indicated in column A on the previous cell number (see *Figure 1b*). Then the student clicks on cell B2 and types a number, for example 86, and selects the "Show Values" option so that the spreadsheet displays numbers instead of cell rules as in *Figure 2a.*

	A	B
1		
2	Choose a Number	86
3	Add four	90
4	Multiply by three	270
5	Divide by two	135
6	Subtract three	132
7		

Figure 2a

	A	B
1		
2	Choose a Number	86
3	Add four	=B2+4
4	Multiply by three	=3*B3
5	Divide by two	=B4/2
6	Subtract three	=B5-3
7		

Figure 2b

The student should now type several different numbers into B2 and observe for each number entered that the spreadsheet automatically calculates the other cell numbers using their respective rules. Following this, the spreadsheet should be used to find, by a 'guess, check and improve' strategy, which number to choose so that the final result is a specified number. Students learn about the behaviour of linear functions provided they think carefully about the 'improve' step. Class discussion is essential here — mindless use of the computer is not instructive.

By selecting the "Show Formulas" option, the spreadsheet display once again allows the student to see the rules they created for each cell number, but this time with some number in B2 as in *Figure 2b*. The chosen number is in B2, but the rule which gives the final number in cell B6 is ***= B5 – 3***.

An obvious question to ask is: What is the rule for the final number in terms of the chosen number? To answer this question, a diagram showing the value of each cell in terms of each prior cell value can be constructed (see *Figure 3*) and this provides an opportunity for some useful discussion about the meaning of algebraic expressions and substitution. In particular, students should see that each cell number can indeed be written in terms of B2.

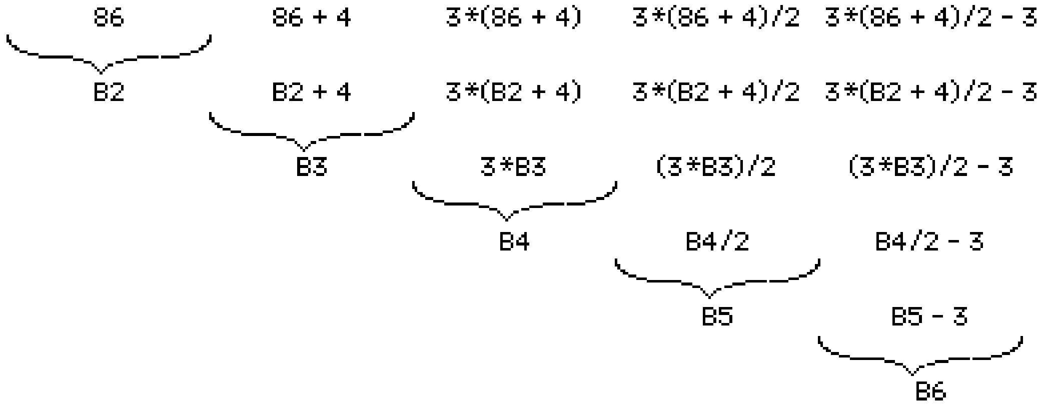

Figure 3

Such an introduction can lead to similar structured activities where the student begins with a chosen number, constructs individual cell rules to give each number produced in turn from a sequence of operations, tries several different values for the chosen number, recognises from the results a simple rule connecting the chosen number and the resulting number, and finally shows that this rule works. This provides a useful stimulus and context for algebraic substitution and simplification of expressions. For example, in the illustration in *Figure 4*, the student should see and demonstrate that although the B6 rule is = ***B5/4***, in terms of B1, it is = ***B1*** + ***1***.

	A	B	C
1	Choose a Number	55	
2	Double it	110	
3	Add one	111	
4	Double it	222	
5	Add two	224	
6	Divide by four	56	
7		The resulting number	

Figure 4

Students should be encouraged to build a sequence of cell rules to ‘backtrack’ to the chosen number and will feel confident that they have done this correctly if, for any number they choose to enter in B1, their sequence of backtrack rules always produces in the final cell, D2, the chosen number (see *Figure 5*).

	A	B	C	D	E
1	Choose a Number	55			
2	Double it	=2*B1	The same?	=D3/2	Half it
3	Add one	=B2+1		=D4-1	Subtract one
4	Double it	=2*B3		=D5/2	Half it
5	Add two	=B4+2		=D6-2	Subtract two
6	Divide by four	=B5/4		=4*B6	Multiply by four
7	The resulting number				

Figure 5

An extension is to challenge students to create their own sequence of operations which will always return the chosen number and then give the related sequence of operations that will backtrack to the chosen number. For the example shown in *Figure 6,* the student would be asked to show why the resulting number in cell B12 always equals the chosen number in cell B2.

	A	B	C	D	E
1					
2	Choose a Number	47		=D5	Final Backtrack Number
3					
4					
5	Add three	=B2+3		=D6-3	Subtract three
6	Double it	=2*B5		=D7/2	Divide by 2
7	Add the Original Number	=B6+B2		=D8-B2	Subtract the Original Number
8	Divide by three	=B7/3		=3*D9	Multiply by three
9	Subtract two	=B8-2		=D12+2	Add two
10					
11					
12	Resulting Number	=B9		=B12	Starting Backtrack Number

Figure 6

Example 2: Modelling a real situation

Spreadsheets can be used to help develop students' concepts of equation and solution and their representation in both numerical and graphical form (Asp 1991). With this knowledge and skill base, students can be taught to create and use spreadsheets to explore problem situations involving these concepts.

For example, the student could construct and use a spreadsheet to explore the question of whether or not LPG should be installed in the family car. To answer this question first requires the identification of the variables which will affect the yearly fuel costs for running a car on each type of fuel. Then follows an investigation of the cumulative cost over a number of years using these cost variables, and this could lead to the construction of a spreadsheet like that shown in *Figure 7*. Changing the value of any one of the input variables allows the student to see, in a dynamic way, the resulting effect on the cumulative costs in the table values and the related graphical representation of the table values. For example, in the EXCEL spreadsheet shown in *Figure 7*, if the initial cost of the LPG installation is increased (cell C7), the student can simultaneously observe the numbers in column G (LPG cumulative cost) increasing, the differences in column H altering and the graph of the LPG cumulative cost function shifting up. Note that the graph shows that the 'break-even' point for these assumed costs occurs after about 3.3 years, and this is confirmed by the values in column H. Furthermore, changing the step and starting values enables the user to hone in on this point. (Note that the model here uses constant fuel costs — a more sophisticated model allowing for, say, annual or even monthly increases in these costs could be developed.)

	A	B	C	D	E	F	G	H
1	INPUT VARIABLES					Cumulative	Cumulative	
2				Step	Year	Petrol Cost	LPG Cost	LPG-Petrol
3	Dist/yr (km/yr)	13000		1	0	0	1500	1500
4		Petrol	LPG		1	842.4	1890	1047.6
5	Econ (L/100 km)	9	12		2	1684.8	2280	595.2
6	Cost ($/L)	0.72	0.25		3	2527.2	2670	142.8
7	Install Cost ($)	0	1500		4	3369.6	3060	-309.6
8					5	4212	3450	-762
9					6	5054.4	3840	-1214.4
10					7	5896.8	4230	-1666.8
11					8	6739.2	4620	-2119.2
12					9	7581.6	5010	-2571.6
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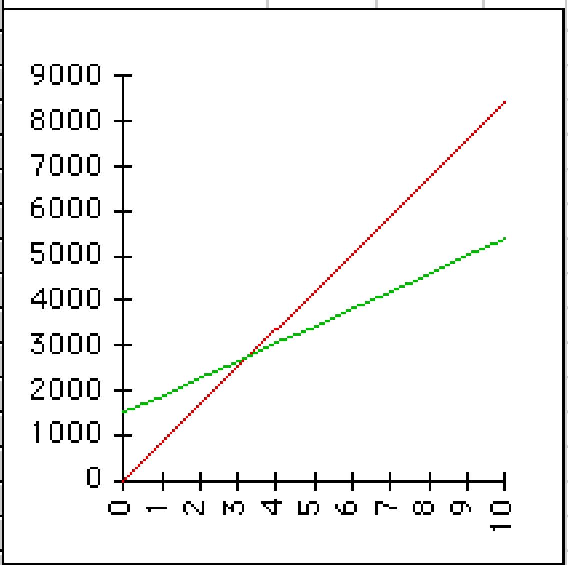


Figure 7

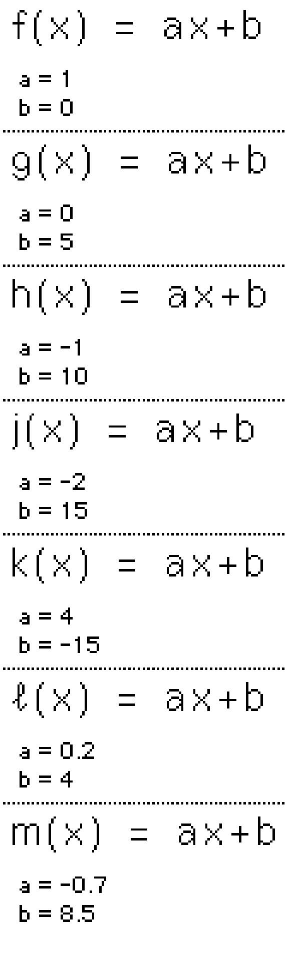
Potential Benefits to the Student

Our earlier research on the use of spreadsheets for linear equation solving (Asp, Dowsey & Stacey, 1992) has given us some encouragement for believing that activities like those illustrated can assist the development of students' understanding of the important algebraic notions of variable, expression and equation. However, it is not known whether 'spreadsheet algebra' with its unique cell symbolisation for variables and operative interpretation of "=" does in fact provide a useful mechanism for learning the usual algebraic notation and manipulative skills. Some researchers hypothesise that the spreadsheet use of "=" encourages students in the belief, often developed in primary school, that "=" means to work something out. For equation solving in algebra it is important that students realise that it means that the two sides are equal, not that one is the answer to the other. Spreadsheet notation may help or it may hinder. Questions such as this one are being investigated in the Technology Enriched Algebra Project.

Graphing Applications

Example 3: Open investigation

In addition to the carefully structured activities presented above, graphing applications and spreadsheets can also be used for open-ended investigation. The flexibility of a graphing application enables a teacher to set tasks to consolidate student learning or to extend more able students. For example, after a unit on graphing linear functions, students might be asked to use a graphing application (e.g. ANUGraph or Capgraph) to draw a set of lines all passing through a specified point such as (5,5). To make the students' use of ANUGraph easy, the teacher can prepare in advance a set of functions with undefined constants and set up an appropriate viewing rectangle. Students can enter values for the constants a and b as shown in *Figure 8a*, view the resulting graphs and change them until the desired orientation is achieved. In doing this students learn how to make a straight line graph pass through a given point. Some students will be unhappy with a result such as that shown in *Figure 8b* which displays little symmetry and unequal angles between the lines. By experimenting to fix this, changing values of a and b, students can get an intuitive feel of the trigonometric relationship between numerical gradient and angle. A line of gradient 2 does not, for example, lie 'halfway between' lines of gradient 1 and 3.



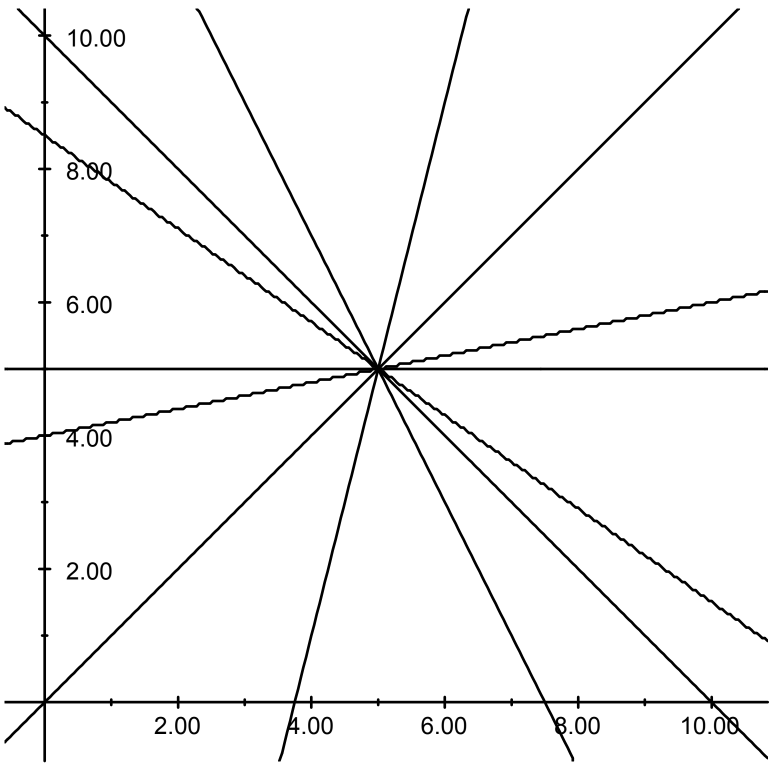
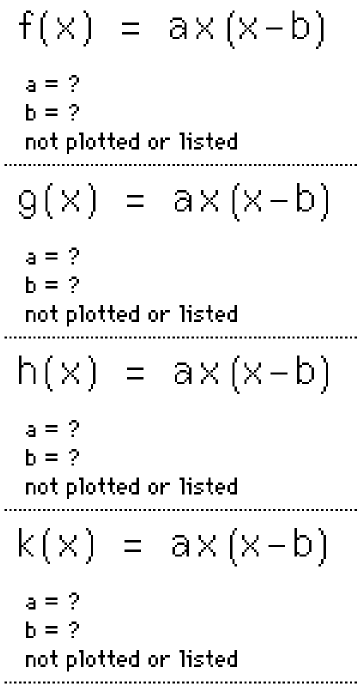


Figure 8a *Figure 8b*

Many variations on this task can be constructed to use functions that the students have been working with. For example, students could be asked to choose constant values for a and b in the functions f, g, h, k, etc. in *Figure 9a* to create the "necklace" of parabolas shown in *Figure 9b*. For some students, the task would be quite different if the functions in *Figure 9a* were given not in factorised form but as $f(x) = ax^2 + bx$, where the intercept is not quite as obvious.



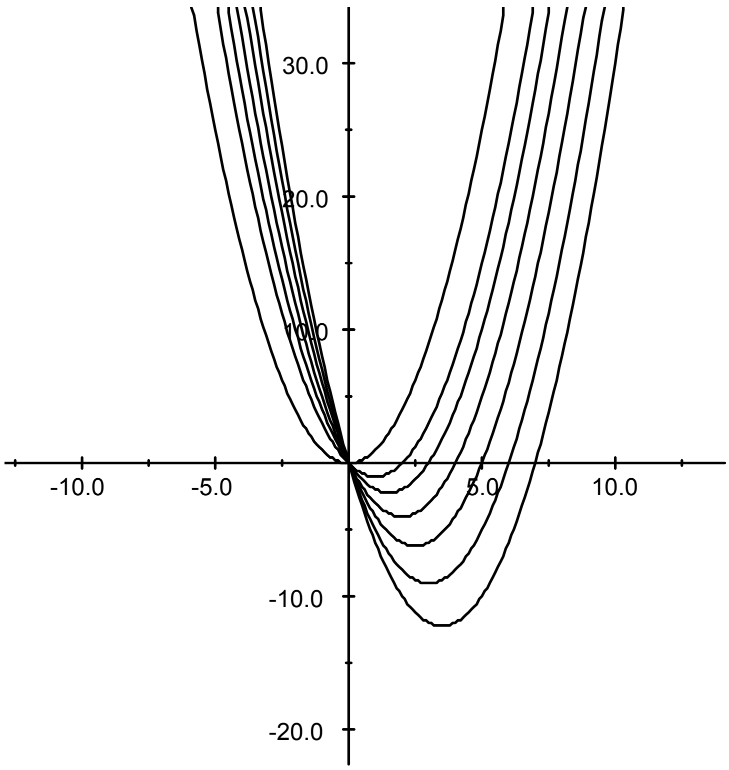


Figure 9a *Figure 9b*

Teachers will be able to invent many other geometric designs that students could try to reproduce by adjusting the constants in other sets of functions. In some instances students may be provided with actual computer output showing co-ordinates and axes but it would often be best to give only the geometric design so that students can choose their own scales and location for designs. This also makes the patterns more attractive.

Teaching with Technology

Sequencing of activities

Where students have had little experience of spreadsheets or graphing applications, we and other researchers (Asp et al, 1992a,b; Sutherland, 1993) have found that there is initial confusion and anxiety as students approach an unfamiliar environment. Clear structure and sequencing needs to be provided at this stage to avoid 25 individuals seeking personal assistance at the same time. If students do not have some prior familiarity with basic computer skills (keyboarding, using a mouse, file access), the first lessons need to present mathematical ideas more slowly as some attention will be diverted to learning about the machines. However, we have found that even novice computer users can get into some carefully sequenced mathematical work from the very beginning.

The importance of teaching

Our observations of classrooms have highlighted several important aspects of teaching with technology. The access which students have to machines is extremely variable. Whereas teachers in a few schools are working with class sets of notebook computers available for every lesson, most have only severely limited regular access or short-term special access such as when another year group is on work-experience. When only short-term access to computers is available, teachers naturally feel that students should use every minute working on the machines. Our classroom observations have shown that then class discussion or direction is often overlooked. Maximum time on the machines does not necessarily equate with maximum learning advantage.

In a British study using spreadsheets with a class of 14 – 15 year olds, Sutherland (1993) observed that teacher behaviour in the first few sessions was critical. She found that students were at first very demanding, wanting to have all their problems solved for them by the teacher. The teacher needs to tread a fine line between giving encouragement to engage with the work to help overcome initial anxieties while insisting that students try to think about the mathematical ideas involved. Discussion with individuals, groups and frequently with the whole class is an important component that should not be dismissed just because the teaching environment is novel. We have seen many students who have worked with the computers mindlessly. For example, their strategy for finding solutions to an equation with a spreadsheet has remained at '*guess, check and try again*' rather than moving on to '*guess, check, look and improve*'. Teachers have to be able to deal with the management role of supervising work on the machines whilst still retaining their role as teacher, drawing students' attention to the important ideas they are working with.

Support for the use of computers and calculators in teaching mathematics is evident at local, state, national and international levels. Basic new tools for doing mathematics, such as spreadsheets and graphing applications, are now easy to use and widely available but important questions of teaching methodology, organisation and management arise. Over the next few years, their rightful place in the teaching of mathematics will become clearer.

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