THE EFFECTS ON STUDENTS' PROBLEM SOLVING BEHAVIOUR OF LONG-TERM TEACHING THROUGH A PROBLEM SOLVING APPROACH.

Kaye Stacey[1]

Institute of Education, University of Melbourne

Abstract

Students from two Year 9 classes at an Australian high school were interviewed as they worked on various mathematical problem solving questions. One class had for three years been taught by a teacher very committed to teaching through a problem solving approach and to demonstrating the everyday usefulness of mathematical ideas. The other class had received traditional instruction, with only occasional problem solving tasks given mainly for amusement or as a "fill-in." Three principal differences in their approaches to the questions were observed. Students from the class which emphasised problem solving worked more deliberately and kept helpful written records of their work. They were noticeably less prone to close quickly on an answer by combining numbers in the question in a superficial way. Instead they were more likely to use a guess and check strategy effectively.

Introduction

Although there is a substantial body of research into teaching mathematical problem solving, almost all of the research evaluates the success of programs designed outside the school (sometimes by a teacher-researcher) and therefore taught to children as something special, not as a routine part of the curriculum (see, for example, Charles and Lester, 1984; Groves and Stacey, 1988; Isaacs, 1987; Kantowski, 1977; Stacey, 1989). Until recently, there was little alternative to this: instruction-related research into mathematical problem solving had to begin in curriculum development by researchers, because the teaching of problem solving was not widespread in schools. However, recent changes in curriculum in Victoria (Australia) have encouraged many teachers to develop their own "problem solving approach " to teaching or special programs in their classrooms. This paper examines the effects of one such teacher-devised mathematics program oriented to improving students' problem solving performance by comparing the problem solving behaviour of those students with that of similar students who had had "traditional instruction".

There are important reasons for looking at problem solving programs which have been constructed and delivered by ordinary teachers, which will, after all, be the programs which almost all children receive. These programs can

[1] This research was supported by a grant from the Australian Research Council. I wish to thank Ian Isaacs, Francesca Calati, Cecilia Del Beato and Mollie MacGregor for many valuable discussions and their careful assistance with data collection and analysis and the teachers and students who participated so willingly in the study.

be expected to differ in a number of ways from the researcher-designed programs which have previously been studied. The goals of a teacher may be different to those of a researcher who is steeped in theories of problem solving. Because of their intimate involvement in the dynamics of the classroom, teachers will also choose goals which they see as achievable in that setting and teaching methods which will enable them to maintain a working atmosphere. Teaching approaches that spring from a teacher's own commitment are the ones that will be carried through consistently over a long time and therefore have a better chance of making a tangible difference.

Selection of Subjects

Stratified samples of students from two classes of Year 9 students at a suburban high school were interviewed. The two classes, known as 9P and 9N, were chosen because they were apparently similar mixed-ability classes from the same cohort of the same school yet with quite different instructional histories. For their three years at the school, the classes had been kept intact as mathematics groups. Class 9N had been taught for three years by teachers who use a traditional approach working from the textbook or worksheets which explain how each type of question is done. Although the class has occasionally been given "problem solving" tasks to do, there has been no regular part of the curriculum devoted to it. One girl , interviewed as she worked on a variant of a missionaries and cannibals crossing a river problem, commented that she had done something like it before "as a sort of joke".

The other class, denoted here as 9P, had been taught for the same three years, by one teacher, here called Frank. During his three years at the school, Frank had become acknowledged as the initiator of problem solving activity in the school and 9P was the class where he trialled his ideas. Frank is particularly concerned with improving the problem solving skills of his students in the widest possible sense and to make them able to deal with everyday applications of mathematics. This, for Frank, is the most important aspect of his teaching. He dates the strong emphasis on problem solving in his teaching from a staffroom conversation, when 9P were his year 7 class.

> *"I was sitting in the staffroom one day when an experienced maths teacher asked me how the classes were going. I explained that I wasn't enjoying it, that the algebra was a real struggle and that the kids were not getting anything out if it. "Well, don't do it", she said. "Teach something useful". That's when the penny dropped. I'm supposed to be some sort of professional. I should be able to make those decisions. Not that I'm not going to do algebra, but I try and direct the course to real life applications - what the kids really need".*

From that time on, Frank began to introduce special problem solving activities in each of his classes and also began to spend more time on the everyday applications of the mathematical content he was teaching. Frank likes to introduce each new topic with a practical example, which he lets students explore in their own ways. He persists with the context , gradually drawing the mathematics out from it and introducing the abstract, general language along with the context-specific words. For example, we observed a lesson where he introduced some ideas about linear equations to his Year 9 class by asking the class to determine a taxi's flag-fall and the charge per kilometre given the fares for various journeys. He examined students' naive approaches sympathetically , eventually adding the graphical and algebraic approach. The class then went on to practise plotting graphs, reading intercepts and interpolating and extrapolating values. For most of the lesson, he used the context-specific terms, rather than abstract terms such y-intercept or gradient. Frank is keen to develop in students a questioning attitude and a resourcefulness so that "*if they can't solve it one way, they should be able to solve it another way*".

Although Frank adopts the "problem solving approach" described above to most of his teaching, he also gave 9P work on "unfamiliar" problems (examples are given in Figure 2) which could be solved with strategies such as drawing a diagram and guess and check. Frank's students were generally positive about this particular component of their work. When we observed one of these lessons, Frank handed out a worksheet of problems similar in length and difficulty to those shown in Figure 2 and suggested that students to work on it in pairs. He helped students as they worked and presented solutions on the board, with interaction from the class. No alternative solutions were discussed and the problems were not extended in any way. "Guess and check" and " make a table" were the only strategies mentioned in this lesson, although he does also stress "drawing a diagram" and "looking for patterns". Frank's program does not contain features broadly associated with "looking back" or increasing meta-cognitive awareness that are widely recommended in the pedagogical literature (e.g. Lester, 1989).

Selection of Tasks and Interview Methods

In order to observe differences in approach to problem solving, we interviewed and audio-taped a stratified sample of 9 of the 24 students from each of 9P and 9N on two occasions, firstly as they worked on the Plumber's Fees (see Figure 1) and then on a selection of 9 questions provided by Frank as being typical of the tasks he used in his problem solving component (see Figure 2). Some prompting was given when students were not making any progress. To encourage students to think aloud, two or three students, roughly matched in ability, were interviewed together: in some groups, students interacted strongly, others not. Frank's problem solving questions were definitely familiar to all his students (and to some students from 9N) but there was no reason to believe that one class was better prepared for the Plumber's Fees than the other as both had worked on linear equations recently. The question had been selected independently and before Frank's lesson on taxi fares was observed.

Figure 1: The Plumber's Fees.

A plumber's fee is made up of a fixed charge for making a visit as well as a charge depending on how long the job takes. For a job taking 15 minutes, her total charge is $28. For a job taking 40 minutes, her total charge is $58. And for a job lasting one hour, her total charge is $82.

(i) What would be her total charge for a job lasting 70 minutes?

(ii) What is the fixed fee charged by the plumber?

(iii) What is her charge for each minute of work?

Figure 2: Examples of Frank's questions with typical solutions from 9P

1. Which number between 1 and 150 when multiplied by itself produces the closest number to 300?
2. At 6.30am the first two people arrived at the Melbourne Cricket Ground to buy tickets for the grand final. Every 25 minutes after that, 3 more than the number of people already present arrived to get in line. How many people were in the line at 9.00am?

Results

As would be expected from their greater familiarity with his questions, Frank's students obtained more correct answers to his problems than did students from 9N. The interviewers' impression is also that the 9P students needed less prompting to solve the Plumber's Fees question. However, the principal purpose of the study was to observe differences in behaviour and the three features described below stood out.

(a) Methods of Recording.

There was an obvious difference in the way that students in 9P and 9N approached the problem solving tasks. Whereas all but one or two of the 9P students worked fairly methodically, students from 9N rushed and guessed, frequently punching the buttons of their calculators furiously. Despite the fact that ample scrap paper and graph paper was supplied , only one student from class 9N wrote anything at all (except for the answers) during the fifty minutes they worked on Frank's questions. The lack of an organised written record was a clear cause of lack of success for 9N on several occasions. In contrast, students in class 9P kept a much more helpful written record at both problem solving sessions, (although their records were neither neat nor sequential) and moreover, they used their recording as a guide to help them decide what to do next.

(b) Grabbing at surface relationships

As explained above, the students of 9N attacked problems in a rush. This behaviour seems to be a symptom of their relentlessly trying ways of manipulating the numbers in the question to come up with an appropriate answer. This tendency to grab at surface relationships rather than to explore the problem was the major mathematical difference between class 9P and class 9N. With the Plumber's Fees problem, all four 9N groups arrived at a (wrong) answer within a few minutes. In three of these groups, students quickly agreed that the answer was $114, which is obtained by noticing that 70 can be made up as 40 + 2 x 15 and so deducing that the price for 70 minutes is made up from the price for 40 minutes plus twice the price for 15 minutes. This method seeks the wanted quantity from a simple combination of the data in given in the question. Only one of the 9N students saw his first answer of $114 as a hypothesis, rather than a definite answer. He went on to test the consistency of his hypothesis with the data provided. For all the others, this early closure on a wrong solution clearly blocked their subsequent thinking about the question, even though they were told that the initial answer was wrong.

The solutions from students in 9P began quite differently. No student suggested $114. Two students were able to solve the problem quite directly: one with a graph and another tabulated differences and found a complicated but correct number pattern. The others had a period of exploring the problem (either guessing a fixed charge, a charge per minute or calculating ratios of prices to times) but they did not close quickly on an answer. Five students used a "guess and check" strategy and some features of this behaviour will be taken up later.

A similar tendency to grab at surface relationships was in evidence in the solutions by 9N to Frank's questions. For example, the wording of Question 2 is particularly hard to interpret. Most of the students in 9P (and two of the three 9N students who were finally prompted to make a table) were unable to co-ordinate all aspects of the question and consequently obtained (wrong) answers of 20 or 77. A typical solution from 9P is shown in Figure

2. Students from the below average group of 9N exhibited quite different "grabbing relationships" behaviour. Their first, almost immediate, solution was 125 people - "*every 25 mins after 6.30, 5 (3 more than 2) people arrive so 25 mins times 5 people is 125 people"*. One of the three students disagreed with this and set about finding the number of 25 minute intervals in the two and a half hour period from 6.30 to 9.00. This was done by calculating 2.30/25 (= 0.092), 2.30/0.25 , 150/0.25 and 150/ 0.992 (presumably an error for 0.092). The group then multiplied 9.2 by 5 which gave 46, their agreed answer to the question.

(c) Use of "guess and check"

Along with the use of tables and charts, "guess and check" is the principal strategy emphasised in Frank's approach to problem solving. The wording of Frank's question 1 (see Figure 2) for example, perhaps triggers a "guess and check" approach and reflects Frank's emphasis. All but one of the 15 students who tackled this at interview, solved it by "guess and check", choosing whole numbers and multiplying them by themselves on the calculator until the answer was nearly 300. Apart from the inefficiency caused by absence of written records of the trials by 9N, there was no difference observed in the effectiveness with which the two classes of students were able to reach an integral solution.

At the simple level required by Question 1, guess and check is an intuitive strategy available to all, but the Plumber's Fees brought out substantial differences between the groups. Table 3 gives the numbers of students from each class who attempted The Plumber's Fees and the three of Frank's nine questions which were amenable to a guess and check strategy. Also given in Table 3 are the numbers who used a guess and check strategy either largely successfully or unsuccessfully.

The lack of spontaneous choice of guess and check by 9N is probably explained by their urge to close quickly on an answer. Since superficially manipulating numbers is "successful" (in that answers were produced), they did not spontaneously seek any other methods and the interviewers' preferred prompts were graphical and algebraic methods. Frank's students (9P) instead began by exploring the question (by calculating differences, ratios or guessing) and in this way, five of them were lead into a guess and check solution. The four successful students used less than three guesses each to find the fixed charge or rate. The two pairs of 9N students who unsuccessfully used guess and check after prompting, failed. Onepair did not appreciate the logical structure of a guess and check solution, the other lacked an adequate written record. In order to find the fixed charge by guess and check, students have to realise that the resulting charge per minute will be constant at the three data points. Without this guiding understanding of structure, the method makes no sense. In contrast, the two instances of failure of guess and check by students of 9P were more technical in nature. One student was unable to cope with the associated awkward calculations as she sought the non-integral charge per minute in Plumber's Fees and another did not recognise all the constraints in Question 9.

Table 3: Use of guess and check strategies

		9P			9N		
	Tries	Succ	Unsucc	Tries	Succ	Unsucc	
Question 1 (Frank's)	6	5	0		9	9	0
Question 6 (Frank's)	3	3	0		1	0	0
Question 9 (Frank's)	4	3	1		1	1	0
Plumber's Fees	9	4	1		8	0	4*

The table gives the number of students from the group who attempted each question (Tries) and the number who used a guess and check strategy largely successfully (Succ) or unsuccessfully (Unsucc) for each question.
* Guess and check strategy prompted by interviewer when other methods did not seem promising.

Conclusion

The results shown above indicate several ways in which the students exposed to three years of teaching oriented towards problem solving have taken on some of the characteristics of able problem solvers. Forty years ago, Bloom and Broder (1950) described unsuccessful problem solvers as spending little time considering questions, but choosing answers on the basis of a few clues. The contrast between the initial behaviour of students from 9P and 9N on the Plumber's question illustrates this behaviour. The students with the "traditional" teaching grabbed at superficial relationships in the problem, manipulating the numbers to get a quick answer. Students from the class which emphasised problem solving were more prepared to explore the problems and were more organised, having come to use the tool skills (principally use of labelled tables) which have been modelled for them by their teacher. They rushed at answers less, they wrote more and they were more deliberate in their calculations.

Both groups of students spontaneously used a guess and check strategy when it was very clear how this should be done, but only students from the problem solving class used it successfully when it was not immediately obvious what to guess and what constraints could be used to check it. The most important gain seemed to be their willingness to explore the problem, rather than try to manipulate numbers in the question to get a quick answer.

Familiarity with guess and check has clearly strengthened the ability of students from the problem solving class to solve problems, and has substantially contributed to the resourcefulness that Frank seeks. However, their propensity to use this intuitive method is of concern. A better balance needs to be maintained so that the strengthening of the guess and check strategy does not inhibit their looking for the more powerful mathematical relationships that underlie problems.

References

Bloom, Benjamin S. and Broder, Lois (1950) Problem Solving Processes of College Students. Chicago: University of Chicago Press.

Charles, Randall. I. and Lester, Frank K. (1984) An Evaluation of A Process-Oriented Instructional Program in Mathematical Problem Solving in Grades 5 and 7. Journal for Research In Mathematics Education, 15 (1), pp15 - 34.

Groves, Susie & Kaye Stacey (1988) Curriculum Development in Problem Solving. In H. Burkhardt, S. Groves, A. Schoenfeld & K. Stacey (eds) Problem Solving: A World View, , Shell Centre For Mathematical Education, Nottingham, England, pp. 199 - 206.

Kantowski, M.G. (1977) Processes involved in mathematical problem solving. Journal of Research in Mathematics Education, 8 (3), pp 163 - 180.

Isaacs, Ian (1987) The effects of extended teaching in problem solving styles on the mathematical achievement of secondary school students in public examinations. In Ong Sit Tui (ed) Proceedings of the Fourth Southeast Asian Conference on Mathematical Education, Singapore: Institute of Education, pp 162 - 167.

Lester, Frank K. (1989) Reflections about Mathematical Problem Solving Research. In R.I. Charles and E. A. Silver (eds) The Teaching and Assessing of Mathematical Problem Solving, Laurence Erlbaum, U.S.A.

Stacey, Kaye (1989) Finding and Using Patterns in Linear Generalising Problems, Educational Studies in Mathematics, 20 (2) , pp 147 - 164.

Stacey, Kaye & Susie Groves. (1985) Strategies For Problem Solving, Latitude Publications, Melbourne.