Linking Application and Acquisition of Mathematical Ideas Through Open Problem Solving

Kaye Stacey
Institute of Education
University of Melbourne

Abstract

There have been substantial advances in teaching using open problem solving in Australia in the last decade, brought about by a desire for improving learning experiences and new curricula and assessment. The term "problem solving" encompasses both real world applications of mathematics and pure mathematical investigations. Experience of solving problems is essential, as is active reflection on that experience. Students' ability to solve problems can be improved by the acquisition of good habits of working. The successful implementation of heuristic strategies is difficult to teach, but the development of a classroom vocabulary for discussing process aspects contributes to awareness of mathematical thinking. Two contrasting problem solving programs have both assisted students to consider problems more fully, to be more prepared to explore and understand, and to be less prone to superficially manipulating the numbers in the question.

AUTHOR'S ADDRESS

Dr Kaye Stacey
Institute of Education
University of Melbourne
Grattan Street
Parkville 3052
Victoria, Australia

1. Problem solving in Australian Schools

Across Australia during the eighties, the teaching of problem solving and modelling has expanded from a fringe activity which interested a small number of teachers to a formally recognised part of all mathematics curricula. Early in the eighties, problem solving became prominent in the curriculum aims for students in the compulsory years of schooling (5 to 16 years old) and now, at the turn of the decade, it is being incorporated into the curricula for the final years of schooling. The growth of the problem solving movement in Australian mathematics education and in official curricula has been documented from the sixties to the nineties (Mortlock,1988; Stacey and Groves,1984; Groves and Stacey, 1988; Stacey and Groves, 1990).

As elsewhere in the world, in Australian mathematics education "problem solving" is a many facetted term. An examination of curriculum documents, classroom practice and assessment methods reveals a wide variety of interpretations of what this much publicised "problem solving" might be. There are differences between individuals, schools and the eight separate governments which control schooling across Australia. Some see it as a process underpinning all mathematical tasks, others see a separate activity which complements the learning of skills and concepts with the learning of general strategies. Others advocate it as a method of teaching those very same skills and concepts. In the sense of Schroeder and Lester (1989), there are advocates of teaching for, about and via problem solving. Some use "problem solving" as a label which refers almost exclusively to bringing more real world applications into the classroom, whilst others use the terms "modelling" and "applications" in this sense and restrict the meaning of the phrase "problem solving" to pure mathematical investigations where students themselves often do a large amount of the problem posing. My own interpretation embraces both of these real world and pure mathematical aspects.

Perhaps the strongest commonality of definition has been in the negative: problem solving is generally not seen as taking place in the most common form of lesson which still consists of a short exposition of theory and examples, followed by practice of many similar exercises. Whilst problem solving, whatever it might be, remained a fringe activity, a wide variety of interpretations were easily embraced, but the imminent assessment of problem solving in the final years of schooling may soon impose a *de facto* definition within each state.

The recent history of curriculum reform has taught us that it is an important step to change syllabus descriptions and official aims, but it is quite another thing to change the learning and teaching that children experience in average schools. During the last decade, steadily increasing numbers of individual schools and teachers have begun to respond to the call for emphasis on problem solving and applications - often because of pressure from curriculum statements but also because of a widespread desire to improve the quality of mathematical experiences offered to pupils. In a survey conducted by the author in a representative sample of 18 Melbourne schools in 1988, over half the teachers reported that they were actively working on increasing the emphasis on problem solving in the curriculum at their school. Moreover, at only one school did the teachers report working on a change that was not directly to do with problem solving. The even more recent changes in senior school curriculum and assessment have accelerated this process. For example, in Victoria, one of the Australian states, it is now compulsory for teachers to spend between 20% and 60% of mathematics time in the final two years of schooling on problem solving and modelling. On the one hand, this decree has caused panic, resentment, confusion and some nifty reclassification of unchanged routine work as problem solving. But, on the other hand, it has also brought many teachers across the state to engage in open problem solving for the first time since their student days. In this way, many of them are rediscovering the forgotten pleasures of mathematical thinking that originally attracted them to their careers.

2. Aims for Open Problem Solving

Problems, in the phrase of Halmos (1980) are the heart of mathematics. Open problem solving reflects the twin sources of mathematics: its intrinsic interest and its utility. Because of its centrality, problem solving cannot be easily defined or delineated; the goals which engender it are broad; the teaching methods which can be used to accomplish them are diverse. It seems essential that in any task described as "problem solving", the solver should encounter a significant degree of unfamiliarity. In the classroom, problem solving activities are characterised by a freedom (indeed a requirement) for students to follow their own paths to a solution, rather than following a routine or procedure set by the teacher. Problem solving activities can offer experience of mathematics as a creative endeavour to pupils of all ability levels and ages. When the application of mathematics to a real situation is involved, problem solving tasks should generally require the formulating of a suitable mathematical model and interpreting and evaluating the results, rather than only the practising of mathematical techniques for solving the model. The problem solving tasks that most interest me are those where the use of mathematical ideas gives significant insight to understanding the problem, however it is important to recognise that for realistic problems of everyday life, often organisational, social and communication skills are more important than the mathematical analysis.

For me, the most important goals are to encourage a deeper understanding of mathematical ideas, to foster attitudes which empower students to use mathematics and to build knowledge, habits and strategies which help students employ their mathematical thinking wherever it may be useful. These are described in more detail below.

2.1 Learning and Using Mathematical Ideas.

Developing mathematical concepts and learning how to use them are closely intertwined processes. Some problem solving activities should offer students experience in using the mathematical concepts they have learnt, other problem solving activities should contribute to the acquisition of new concepts. One learns mathematics largely so that it can be used to solve problems, but one also grapples with unfamiliar problems in order to learn mathematics.

A problem solving approach can introduce mathematical methods in a way which assists students to see the reasons why the ideas are important and makes their uses apparent. By starting with a familiar context which illustrates a new mathematical idea, students can appreciate right from the start what a new idea is for. Simultaneously, they build intuitive mental models to help with their interpretation of the abstractions involved. For example, work on linear equations can begin with a problem which requires students to investigate the pricing structures of one of the many services (such as telephone bills, public transport fares, car rentals or tradesmens' fees) where the cost is based on a fixed charge added to a charge depending on usage (such as how many calls made, kilometres travelled or minutes of work). Suitable starting questions are to predict a cost based on other known costs or to choose between alternative services with different fixed charges and rates. Students who are allowed to explore these questions in their own ways, can gradually disclose the underlying mathematical structure for themselves. Having grappled with the problem in their own ways, they can appreciate the usefulness of the mathematical techniques that they are later taught. Exploration of questions related to the real situation builds a strong intuitive mental model on which the abstract concepts (such as the equation of a linear relationship, y-intercept or gradient) can be established.

As well as being used to introduce mathematical ideas so that their uses are apparent, open problem solving has an essential role to play in assisting students to recognise situations where mathematical procedures and concepts are relevant. Most mathematics teaching leaves students with procedural knowledge that is very tightly bound to specific triggering stimuli. Given a quadratic expression, students will automatically factorise; given a geometric series, they will automatically use the formula and find the sum; a problem about a man rowing a boat up a river will automatically summon ideas about adding and subtracting current strengths from rowing speeds. These are all features of "expert" behaviour, but for many students it is only these specific triggers that invoke mathematical ideas. To be able to use their mathematical knowledge in anything other than textbook exercises, they need regular opportunities for engaging with problems where the appropriate mathematical techniques are *not* triggered by a standard context, a chapter heading or too much guidance from the teacher. Problem solving in this sense cannot be neatly cross-referenced to a content-based curriculum, because its very essence is that students must expect to call upon anything they know to assist in their solutions.

2.2 Attitudes which empower students to use what they know.

Rather than thinking there is one right way to approach a problem (and additionally that this way will be almost immediately apparent), good problem solvers need to develop attitudes of resourcefulness, flexibility, self-reliance and confidence. Teaching which concentrates mainly on practising routine methods for solving standard problems seems to reduce students' resourcefulness rather than enhance it. In addition to often giving students problem solving experiences where resourcefulness is demanded, frequent discussion of alternative approaches is an excellent way to encourage them to develop resourcefulness and flexibility of approach. Pupils who cannot solve a problem one way, should be encouraged to try it in another way, not just shown how to do the problem.

To build self confidence, the classroom atmosphere needs to maintain a delicate balance between being supportive and challenging. Students who are stuck need help but not answers at that stage. Group work is useful to encourage a willingness to tackle problems and it provides a natural context for students to talk about and justify their methods, thereby becoming more aware of their own thinking. Since emotions such as panic and despondency are quite often experienced by students tackling hard problems, teachers need to discuss these feelings, suggesting simple steps to take in these circumstances. For example, *making a list of what you know* is an easy starting strategy which overcomes the starkness of a blank sheet of paper, whilst *explaining what you have done to a friend* is a good way to check your work and summarise what you have found out so far.

2.3 Helpful habits and strategies

One of the easiest ways in which to improve childrens' problem solving is to make them aware of some simple good habits for organising their solutions to problems. Children very frequently keep no records of trials that they make, so that they learn nothing from them and have no possibility of checking their work. When they do write things down it is often completely disorganised, so that they are unable to see regularities and patterns in the data. They also very often start working on questions that they have not completely understood. For these reasons, students from age nine should be encouraged to keep a written record of their work, and be shown how to record data in labelled tables or diagrams and how to organise information systematically. Various strategies for careful reading (such as re-write the question in your own words, underline important information) are frequently taught even in the most traditional classrooms and these also seem to pay quick dividends.

The teaching of other mathematical Polya-type heuristics pays fewer dividends, at least in the short term. Whilst I do not strongly advocate systematic teaching about phases of problem solving or of any strategies beyond *look for and exploit pattern* and *guess, check and improve,* class discussion of heuristics does contribute to the development of a classroom vocabulary about the process of doing mathematics. I see this as particularly valuable - having words to talk about and reflect on mathematical thinking is important for coming to understand it.

2.4 Problems before, during and after learning a concept.

Some problem solving work should be directed to looking forward to new concepts, which will not formally taught for some months or years. The research into students' misconceptions show us how long it takes for concepts to grow - solving an engaging problem can plant seeds in the mind, so that students slowly prepare good mental models for important mathematical ideas such as ratio, infinity or statistical variability. This form of problem solving is particularly important for younger children (Groves and Stacey, 1990)

Some problem solving work should be directed to the topic of the moment - using a problem is a good way to introduce a new concept or skill, although often not such a good way to practise it. As a general rule, all topics should be introduced from an application or a context and these applications should be revisited throughout the learning sequence. It is important that students know what new power a new mathematical skill actually gives them.

Some problem solving activities should be given where students have to use anything that they know to solve the problem. This develops resourcefulness and gives students practice in recognising the circumstances in which their knowledge can be used. This aspect of problem solving is really important, although it will never fit neatly into a content-based curriculum. It is particularly important for older students, who generally develop a wide gap between what they know and what they know how to use.

3. Teaching Problem Solving

Three ingredients are essential in any program designed to improve students' problem solving performance:

experience in tackling non-routine problems

a structure which assists students to reflect upon these experiences and

exposure to simple problem solving strategies and good habits.

The experience of working on challenging problems is clearly the single most important pre-requisite for improving problem solving. Yet, as is stressed by Mason, Burton and Stacey (1982), experience is not sufficient. People do not learn just by doing, but by thinking about what they do. Encouraging students to reflect productively on their work is a crucial part of the teacher's role and probably also the most difficult. In order to illustrate how the three ingredients of experience, reflection and strategies guide the teaching of problem solving, I have selected a question which many readers may have used in their own teaching. In the course of developing materials for teaching problem solving (such as Stacey and Groves (1985) and Stacey and Southwell (1983)) I have observed many children solving variations of combinatorial and geometric versions of the problem of the mystic rose: how many lines must be drawn to make a *mystic rose* by joining every pair of a set of points arranged evenly around a circle?

3.1 An example

The first point to note is that even though this is very often regarded as a typical problem solving question, the attribute of being "problem solving" does not lie in the task or question, but in the manner in which the question is used in the classroom. The often expressed desire of teachers for a vast supply of good questions so that they can get started on problem solving is misplaced. When Susie Groves and I first began to offer a course on problem solving for teacher education students, we started with a huge bank of interesting problems. But our students flitted from one question to another without seriously coming to grips with any, learning a few useful tricks but little about mathematical thinking or mathematical structure. As we became more skilled in teaching problem solving and clarified our goals, we gradually used fewer and fewer questions, but sought to extract more and more learning from them.

In a typical class of twelve year olds working on an 8 point mystic rose question, usually approximately one third of the class begin by proposing that there are 8 x 7 lines, as there are 8 points each of which is joined to the other 7. When they find out that this answer of 56 is not right, they completely abandon their first idea and generally follow the other two thirds of the class who are drawing diagrams and counting the number of lines. Even when discussing this intuitive approach, the teacher can emphasise a number of helpful habits: drawing systematically so that no lines are missed; using a different coloured pencil at each point to assist with the later counting, writing down subtotals as the counting proceeds. As they count, some students will notice patterns in the numbers of lines they are adding. The number patterns reflect the systems they have used for drawing the rose. Most students draw it point by point (so that 7 lines leave the first point , 6 leave the next and so on), but there are other common ways which lead to other number patterns.

If this is as far as the teacher takes this problem, with students finishing off with a statement that the answer is 28 perhaps illustrated with a diagram, then I believe that most of the benefits of using the question have not been accessed. Firstly, it is a magic moment when those students who entirely abandoned their first 8 x 7 approach realise how nearly right they were. Modifying, rather than abandoning, an idea is an important problem solving strategy and it is from noticing and reflecting on significant experiences such as this that students best learn heuristics. They need to be shown how they could have explored the situation (perhaps by looking at smaller cases where the right answer is easy to find) to learn how to modify their first good idea rather than throw it away.

Secondly, students must have an opportunity to re-use what they have found out, so we give a larger mystic rose, say with 20 points . Drawing and counting now is a nearly unfeasible strategy, unless a very systematic approach is followed. With an even larger rose (say 100 points), students are forced to look at the details of the number pattern - they all notice that the numbers in the sum decrease, but don't all know that the series starts at one less than the number of points and goes to zero and they don't know why. Soon they can be gently pushed to consider better ways than using a calculator to find the sum of the numbers in the pattern, particularly if the next mystic rose has a hundred points. In this way, the extension question of solving the mystic rose problem for a larger number of points (a "looking forward") is used to stimulate looking back. We all appreciate the value of Polya's advice to "look back", but this type of reflection is not a common feature of school culture. Children at school expect to *do*, not *think*. Therefore, a teacher needs to provide an active context for reflection - extending the problem and writing about what, how and why are the most natural stimuli.

Thirdly, problem solving experiences like this provide what I see as the best possible opportunities for teaching students about proof and proving. As I have argued previously (Stacey and The Manchester Teacher's Group, 1988), teaching about proof suffers in the extreme from answering questions that students have never asked. Just as it is said that a jumper is something a child puts on when its parent is feeling cold, so a proof is something a student sees when a teacher feels the need to justify why. The student does not usually feel this need - the mere fact that the result is being taught is usually enough to dispel all doubt from his or her mind. In open problem solving, students discover and articulate their own conjectures, such as

> "You can find the total number of lines by adding up the numbers starting at one and going to one less than the number of points."
> or " If a number is an even number, you can find the sum of all the numbers up to it by multiplying half that number by one more than that number.

Because it is the student's result, not the teacher's, the student has a real reason for testing it and for finding a reason why it is true. Proving is a way of finding out what is true as well as why it is true.

Finally, students of this age are beginning to encounter algebra and so they need to extend their ideas of and experiences of generalisation. Students struggle with their awkward attempts to articulate their own generalisations in natural language (as the examples above show) and can be delighted and relieved when they are shown some standard algebraic notation. For example, the awkward natural language conjecture above might be re-written by a beginning algebra student as " If n is an even number, you can find the sum of all the numbers up to n by multiplying half of n by n+1." In traditional teaching, students more often see algebraic notation as a burden rather than as a powerful tool, but this problem solving task provides a natural motivation.

In any class which has tackled this problem in an open way, several different methods of counting or calculating the answer will be found by class members. Finding out if these ways are really the same and will always give the same answers provides a natural context for manipulation of algebraic expressions. For example, students may want to know if the two formulae n(n-1)/2 and (n-1)x(n-1)/2 + (n-1)/2 always give the same answers. Other students will have found slightly different formulas for even and odd numbers of points and will need to see that they can both be written in the same way. In this way, algebraic manipulation which is frequently taught as a completely contextless skill comes to be seen as a powerful tool.

Encouraging students to reflect and extend questions in this way is one of the hardest things to do in an ordinary class. In my current observation of the teaching of problem solving in a typical school, even the most committed teacher generally spends only 2 or 3 minutes discussing each question. Students find an answer, by whatever method they can, but they do not look into the underlying mathematical structure. It is clearly counter to the culture of the school for students to explore beyond an initial answer, particularly if their work is not directed by closely definitive questions. Considering different ways to do a question is probably one of the best tools we have for strengthening the flexibility of approach that good problem solvers show, but it is not easy to make a typical class feel that they are achieving something when they find the same answer in yet another way.

3.2 Reality, interest and motivation.

My choice of the mystic rose question above was determined partly by its likely familiarity to the reader, but also because I find that sharing experience of teaching with problems designed as "everyday, real problems" presents special difficulty. Reality is fragile and does not transport easily. With the possible exception of questions which are exclusively about financial calculations, it does not seem possible for any large group of people to agree on whether an isolated question is a "real life problem". Over many years, I have taken note of the questions that have been asked by people around me in the course of their everyday lives. These questions I then incorporate in my teaching and lecturing. Yet on almost every occasion when the opportunity has arisen, a member of the audience will express doubt that such a question would really arise in real life. For example, I was at the house of a friend when an unexpectedly large telephone bill arrived. In the ensuing family discussion, the teenage daughter suggested "we could check it by working out roughly how many local calls we must have made each day". Some time later, I proposed this scenario for use in a class and the teacher wrote back:

> " Is this question a real world problem - I find it a bit contrived. That is one of the interesting things about problem solving - what is a real world problem? Why would anyone want to know that? A lot of it is mental gymnastics, being able to sit down and work it out. I like it, I enjoy it but I don't see it as a real world problem."

As this experience has been repeated time and time again, I have learned and re-learned that we cannot expect real world questions to stay alive when they have been sanitised and pre-packaged for classroom consumption. A sense of reality has to be recreated for each use by active involvement of the participants. Similarly motivation and interest do not reside in questions. It is pointless to seek questions that are *of themselves* interesting. Instead these are human attributes, which spring from the social setting or from a resonance with a special memory, not from the subject matter itself. Teachers make motivation, not special questions.

5. Outcomes of Teaching Problem Solving.

The author has conducted two contrasting studies into the effects of long term exposure to problem solving instruction. In one study (Stacey, 1989), teachers of twelve and thirteen year old students at a wealthy private school followed a special program designed by the researcher for one lesson per week for a year. This program, which is fully described by Stacey and Groves (1985), is designed to provide experience and reflection, organised around a framework of learning about simple strategies and good habits. The second study (Stacey, 1990) investigated a class at a working class school which had been taught from age twelve to fifteen by a young teacher very committed to teaching through a problem solving approach and to demonstrating the everyday usefulness of mathematical ideas. His program, which gradually evolved over the three years, emphasised the real world origins, everyday applications and sensible use of the mathematical techniques on the syllabus. He has constantly struggled (his own word) to find contexts from which to teach new mathematical ideas. The students also spent time regularly on a variety of short "problem solving questions" which were not related to the normal syllabus. Four strategies (look for a pattern, draw a diagram, make a table, use guess and check) were heavily emphasised. The teacher-designed program of the second study contrasted with the researcher-designed program of the first study principally in the greater emphasis it gave to real world applications of new content and the lack of reflection on the problem solving tasks. These tasks were not undertaken for an extended length of time (5 to 10 minutes seemed about the norm) and the teacher's role was only to provide the initial questions, assist with difficulties and show how to get the answers. Very little teacher or student time was devoted to thinking about alternative methods. In each study, the classes were compared with similar classes who had received traditional instruction, with only occasional problem solving tasks given principally for amusement. The questions used to measure the effectiveness of the programs included some questions which required a significant degree of transfer, in that they were not particularly closely related to the questions only the experimental classes had emphasised.

5.1 Recording

Despite the differences in the experimental classes and the programs of instruction they received, two principal differences in behaviour distinguished students in both experimental programs from the control students. Firstly, both groups of experimental students from the problem solving classes worked more deliberately and kept more helpful written records of their work. They used three modes of recording: to extend short-term memory (e.g. for intermediate results in a calculation), to organise information (e.g. by use of labelled tables), and as a guide to help them to review where they were and to decide what to do next. The recording used by control students was generally only in the first of these three modes. The lack of recording in the other two modes was a clear cause of lack of success for the control students on several occasions.

5.2 Grabbing at surface relationships

Secondly, the experimental students were noticeably less prone to close quickly on an answer by combining numbers in the questions in a superficial way. Control students attacked problems in a rush, often writing nothing down except an answer. This behaviour seems to be a symptom of their relentlessly trying ways of manipulating the numbers in the question to come up with an appropriate answer. For example, in the second study, one question asked students to predict the fee a plumber would charge for a job that lasted 70 minutes. The question explained that there was a fixed fee plus a charge which depended on the number of minutes the job took. The fees for jobs lasting 15 minutes, 30 minutes and 40 minutes were given. The four groups of control students interviewed all arrived at a wrong answer within a few minutes. In three of these groups, they noticed that 70 can be made up as 40 + 2 x 15 and so deduced that the price for 70 minutes is made up from the price for 40 minutes plus twice the price for 15 minutes. This method seeks the wanted quantity from a simple combination of the data in given in the question. The solutions from experimental students began quite differently, with no student suggesting this method. Only one of these control students saw this first answer as a conjecture, rather than a definite answer. He went on to test the consistency of his hypothesis with the data provided - a good problem solving technique (Mason, Burton & Stacey, 1982). For all the others, this early closure on a wrong solution clearly blocked their subsequent thinking about the question, even though they were told that the initial answer was wrong. These students displayed the characteristic of unsuccessful problems solvers described over forty years ago by Bloom and Broder (1950), who noted how they spend little time considering questions, but choose answers on the basis of few clues.

Students from the classes which emphasised problem solving were more prepared to explore the problems and were more organised, having come to use the tool skills (principally use of labelled tables) which have been modelled for them by their teachers. They wrote more and used what they wrote as a guide, they rushed at answers less and were more deliberate in their choice of approach. In summary, although there were substantial differences between the two experimental programs, in both cases the behaviour of the students who had been taught with a problem solving approach was more like that of expert problem solvers than was the behaviour of other students from the cohort.

6. Problem solving, mathematical methods and intuitive methods.

One of the most powerful ways in which to improve students' mathematical problem solving is to strengthen their use of intuitive strategies (such as "guess and check", where an estimate of the answer (or part of the answer) is made which is then compared with the constraints imposed by the question, so that a better estimate can be located). Flexibility and resourcefulness in problem solving depend on students having a range of simple "back-up" strategies to use when more "mathematical" methods have failed. Our observations of students during the development of our problem solving program for junior secondary students (Stacey and Groves, 1985) indicated that many students are impeded in their approach to problems because they believe that exploration by trial and error and specialising in the sense of Mason et al (1982), are not permitted in tackling mathematical problems. They expect to be able to look at a problem and choose a standard mathematical method immediately. This attitude becomes more marked as students become older. Any emphasis on non-routine problems, where students are to use any approach they know, will therefore inevitably involve students in unstructured exploration and the use of primitive, intuitive strategies much more so than traditional teaching.

A growth in willingness and confidence to use intuitive strategies has clearly strengthened the ability of students from the problem solving classes to solve problems resourcefully. However, their propensity to use intuitive methods, rather than the more powerful mathematical methods they have been taught, is of concern. For example, I recently examined the solutions of over one hundred sixteen year old students to a "think of a number" question, where students have to find an unknown number by knowing the result when various operations have been performed on it. Not one of these students chose to attempt the question algebraically. Instead, they had all categorised the question as "problem solving" and had therefore all used guess and check (generally successfully) to find the positive solution - no-one found the negative solution. As one student commented " you don't usually solve problems using the mathematics you learn in year eleven". A balance needs to be carefully maintained so that the strengthening of intuitive strategies does not inhibit students from looking for the more powerful mathematical relationships that underlie problems.

6. Implementing problem solving approaches.

A teacher at a non-academic suburban secondary school, who has experimented with various approaches to teaching problem solving over a number of years and has assisted other teachers in his department, summarised teachers' difficulties in implementing problem solving methods this way.

" We think it is important to do because that's where maths is heading, but it is going to take teachers at least two to three years to acquire the skill to teach [a lesson that emphasises problem solving]. They need about three years to be able to go in, clearly define the problem so the kids aren't just sitting there saying they can't do this. When that happens, the teacher ends up doing the problem for them."

During the past decade, we have seen many practical reasons why increasing the emphasis on problem solving is hard to achieve - for example, lack of time for teachers to read and plan and insufficient in-service education seem permanent features of the system. At the time of ICME 5, there was an urgent need for need for good teaching materials (Burkhardt et al, 1988): since that time, Australia has seen a steady stream of good teaching materials and there are good prospects of filling the gaps which are left. Syllabus statements now recognise process objectives, bringing about a great flurry of activity in schools and making urgent the issues of fair assessment that reflects problem solving goals.

Many of the obstacles to creating a truly problem solving environment in schools are to do with the culture of the school. Students have a notion of a good teacher as one who tells them the answers and helps them painlessly through obstacles. Students expect to work following the teachers' preferred methods - flexibility and resourcefulness are not usually required. Therefore, in the name of solving non-routine questions,this culture places us in danger of creating new classes of routine questions, which are to be approached with simple heuristics rather than a traditional formula. The attitude in school that it is *doing* that is important, makes it difficult to create an atmosphere of thinking deeply and reflection.

Despite the recognition of these difficulties, now is a time of optimism that real gains will be made. There is a widespread recognition that mathematics teaching needs to use problems that capture the imagination and stimulate thinking. Computer technology is forcing a re-assessment of the role of routine calculations in mathematics, which is further strengthening ideas of mathematics as a problem solving discipline. Particularly at the primary school level, teaching methodologies that mimic the natural process of learning to speak are fashionable. When applied to mathematics, they allow children to engage, on an informal level, with a wider range of mathematical ideas than was previously the case. The challenge is to bring about widespread changes in classrooms without reducing the creativity and complexity of mathematical thinking to yet another set of routines.

8. References

Bloom, Benjamin S. and Broder, Lois (1950) *Problem Solving Processes of College Students*. Chicago: University of Chicago Press.

Burkhardt, Hugh, Groves, Susie, Schoenfeld, Alan and Stacey, Kaye (Eds.) (1988) *Problem Solving - A World View*, Nottingham, England : Shell Centre for Mathematical Education.

Groves, Susie and Stacey, Kaye. (1988) "Problem Solving: An Annotated Bibliography", in D. Blane and G. Leder (Eds.), *Mathematics Education in Australia: A Selection of Recent Research*, Melbourne, Victoria: Mathematics Education Research Group of Australasia, pp 66 - 119.

Groves, Susie and Stacey, Kaye. (1990) "Problem Solving - A Way of Linking Mathematics to Young Children's Reality", *Australian Journal of Early Childhood*, 15 (1), pp 5 - 11.

Halmos, Paul R. (1980) The Heart of Mathematics, *American Mathematical Monthly*, 87 (7), pp 519 - 524.

Mason, John, Leone Burton and Kaye Stacey (1982) *Thinking Mathematically*, London: Addison Wesley .

Mortlock, Roland. (1988) "Problem Solving - The State of the Art in Australia", in H. Burkhardt, S. Groves, A. Schoenfeld and K. Stacey (Eds.), *Problem Solving - A World View*, Nottingham, England : Shell Centre for Mathematical Education, pp 144 - 150.

Schroeder, Thomas L. and Lester, Frank K. (1989) Developing Understanding in Mathematics via Problem Solving, in *New Directions For Elementary School Mathematics,* eds Paul R. Trafton and Albert P. Shulte, National Council of Teachers of Mathematics, Reston, Va.

Stacey, Kaye (1989) Finding and Using Patterns in Linear Generalising Problems, *Educational Studies in Mathematics*, 20 (2), pp 147 - 164.

Stacey, Kaye (1990) *What does Teaching Problem Solving Teach* ? Paper presented to the 13th Annual Conference of the Mathematics Education Research Group of Australia, Hobart, July, 1990.

Stacey, Kaye and Groves, Susie. (1984) "Problem Solving : People and Projects in Australia", in P. Costello, S. Ferguson, K. Slinn, M. Stephens, D. Trembath and D. Williams (Eds.), *Facets of Australian Mathematics Education*, Blackburn, Victoria: Australian Association of Mathematics Teachers, pp 205 - 209.

Stacey, Kaye and Groves, Susie. (1985) *Strategies For Problem Solving,* Melbourne: Latitude Publications.

Stacey, Kaye and Groves, Susie. (1990) "The Teaching of Applications, Modelling and Problem Solving in Australia: 1984 - 1988", in Mogens Niss, Ian Huntley and Werner Blum (Eds.), *Modelling, Applications and Applied Problem Solving,* Chichester: Ellis Horwood, in press.

Stacey, Kaye and Southwell, Beth (1983) *Teacher Tactics For Problem Solving*, Canberra: Curriculum Development Centre.

Stacey, Kaye and The Manchester Teachers' Group (1988) *Describing, Explaining and Convincing.* Nottingham: Shell Centre for Mathematical Education.