

Fractals, Paths and Approximations

Kaye Stacey¹
Institute of Education
University of Melbourne

"I have never seen such intense, creative and co-operative work in mathematics. In class, there was a great deal of discussion, yet they were all working on their own problems."

Teacher trialling Common Assessment Task One, 1990.

Introduction

The new Mathematics Study Design in Victoria uses four "tasks" to assess Year 12 Mathematics, each of which focuses on a specific aspect of doing mathematics. Details of the new arrangements are given by the Victorian Curriculum and Assessment Board [VCAB] in the Study Design (VCAB, 1989b) and in the curriculum development support material (VCAB, 1989a). One of the four assessment tasks is a project, where students are required to identify an area of concern, apply relevant mathematical ideas to understand the situation and communicate the results in a polished form. The assessment tasks are the same for all students enrolled in a given Mathematics block - hence they are entitled "common assessment tasks" and abbreviated to "CATs".

In 1990, the third set of trials of the use of projects as a form of assessment for Year 12 Mathematics in V.C.E. has been conducted. In 1988, several classes of students enrolled in the existing Year 12 Mathematics subjects completed a project which was closely related to the work they did for the teacher-assessed part of the normal course. Some students chose from several fixed topics related to the growth and decay option of Mathematics A, whilst the Business Mathematics students submitted the project work on specified topics such as starting a small business. The particular focus for 1988 was the development of the set of assessment criteria - the draft first trialled in 1988 has since undergone many modifications.

In 1989, the first real trial began. The project report format was spelled out and appeared with an extensive set of conditions for the conduct of the project in schools. The verification procedure

¹ The opinions expressed here are those of the author and do not necessarily reflect the views of other members of the Setting Panel for CAT1, other Chairpersons of CAT1 Verification Panels or VCAB.

was trialled with nine schools. The assessment criteria were further modified. An evaluation team reported to the Board on the conduct of the project, the verification procedures and the reactions of students and teachers (Symons, 1989). In order to accommodate individual differences and allow students to work independently, setting their own goals, plans and approaches, the CAT projects in 1989 and 1990 were specified only by theme and not by topic, an arrangement that is being continued into 1991. Different project themes are set for the three Mathematics "blocks" and students, who are free to interpret the themes in any reasonable way, must explain how their topic is related to the theme. In 1989, the project theme for the Space and Number block was "periodicity". To help with the interpretation of the theme, the general advice to students suggested about a dozen broad starting points, such as studying calendars, waves, biorhythms, astronomy, economics, and periodic patterns produced by mechanisms like cams in sewing machines. The use of these starting points was not compulsory, but almost all students in fact used them. Conversely, almost all of the starting points were in fact used.

This year, 1990, has seen what is expected to be the last trial. The advice to candidates and schools has been further refined. The projects have again been specified by theme, and elaborated by "starting points". The verification procedure is moving closer to the real thing, with over 40 trial schools grouped into 6 clusters. For 1990, the Space and Number theme contrasted with the openness of "Periodicity" by suggesting a rather more structured investigation of fractals, which have recently received a great deal of media coverage. The Reasoning and Data theme was "Choosing an Optimal Path", a theme which students could interpret in a spatial sense or a more abstract sense as is involved, for example, in critical path analysis. The Change and Approximation theme on errors and approximations was similarly open, with starting points suggesting errors due to round off in arithmetic, errors in measurement as well as methods of approximating particular functions.

At the time of writing, it is too early to report on how students have tackled these themes, although this will be done in the session that accompanies this paper. However, there are a number of general points that have arisen in the previous trials that may be useful for teachers who will be involved in CAT projects in the future. Although this article is concerned with projects done as common assessment tasks, it is important to stress that not all projects used in Mathematics classes should follow this style.

Projects using Mathematics

The project themes have been chosen as mathematical topics such as periodicity, rather than an area of application such as economics or sport in order to emphasise that it is the reasonable use of

mathematics that is to be assessed in the CAT project. Looked at in a broad sense, project work can help students strengthen a wide variety of life skills (such as using information services of local government, organising a team, typing and computer programming). All of these are valuable and have a place in the overall education of all students, but the assessment of the Year 12 Mathematics project does not emphasise them. The appropriate use of relevant mathematics is a requirement for all grades of project. (See Grade descriptors, Figure 1)

GRADE DESCRIPTORS FOR CAT 1: INVESTIGATIVE PROJECT 1990 PILOT PROGRAM

- A.** Clearly defined the investigation and evaluated the conclusion. Demonstrated high level skills of organisation, analysis and evaluation in the conduct of the investigation. Used high level mathematics appropriate to the task with accuracy. Communicated the results succinctly in the specified project format.
- B.** Clearly defined the investigation. Demonstrated skills of organisation, analysis and evaluation in the conduct of the investigation. Used mathematics appropriate to the task with accuracy. Communicated the results clearly in the specified project format.
- C.** Defined the investigation. Demonstrated some facility in the collection and analysis of appropriate information. Used mathematics appropriate to the task. Communicated the results in the specified project format.
- D.** Defined the investigation. Identified and collected appropriate information. Used mathematics relevant to the task. Completed the report in the specified format.
- E.** Stated a project topic relevant to the theme. Identified basic information. Used mathematics relevant to the task. Completed the report in the specified format.

Figure 1 : Grade Descriptors for Project Common Assessment Task in VCE Mathematics

The CAT project is definitely intended to be an investigation which uses mathematics to describe or analyse a particular problem or situation. Not all useful maths projects are like this. Investigations about the role of mathematics in history, science and culture, for example, would be hard to fit into the guidelines for the CAT project, but could be very successful on other occasions.

The introduction of project work and problem solving/modelling provides a new emphasis on communicating mathematical ideas. As the grade for the common assessment task is decided entirely on the written work presented, the ability to write clearly about mathematical ideas is very important. Whilst a high standard of presentation (typing, colouring in, binding etc) almost

certainly disposes the assessors to look favourably on a project, there is no assessment item related to this physical aspect of presentation. Instead, the communication section of the Assessment checklist concentrates on the substantive aspects of the report, including the organisation of the material, adequate definition of symbols used and clarity of aims, account of the investigation and evaluation of the conclusions. In 1989, some students had difficulty following the specified format: teachers may need to explain the main features of an abstract, appendices and the conclusion.

Organisation

In 1989 and 1990, the project CAT had to be completed within four weeks (this limit is to be extended in 1991), with an expected time allocation of 20 hours, approximately half of which is in class. In order for students to complete the project within this time, it is very important that they work efficiently. In 1989, many students wrote in their reports of their difficulties in getting started on a project; some did not even settle down to a topic until the fourth week, which meant that the teacher was unable to see a reasonable draft of the project before it was handed in. Collecting data was one of the major hold-ups. Many students went to great lengths to obtain data which was not available (for example, AIDS statistics for the years before the disease had been identified or a breakdown of national statistics for a local region). Many relied upon other people to supply data which never turned up. The final result of all this was that many students spent a disproportionate amount of time collecting or waiting for data and too little time analysing it. Whilst these are inevitable problems associated with research, it was clear that many of the problems could have been avoided by using reference publications available in almost any library - the yearbooks for Australia and for each state, for example, contain a wealth of information suitable for projects in probability and statistics.

Extended, Independent Work.

The VCAB study design identifies the project work requirement as being met by extended work, done independently. On both the attributes of extent and independence, the CAT project is at the end of the scale - the projects that teachers use elsewhere may profitably vary greatly in the amount of structure imposed by the teacher and in demand time. For example, Year 9 students may be asked to plan the re-decoration of a house or room within a fixed budget, and the content of the report may be quite closely prescribed; scale drawing of house, list of items to be purchased, calculations showing amount of floor covering and paint to be purchased etc. By providing the

overall structure, the teacher is able to focus students' attention on the desired material and the relatively uniform product is more easily graded by the teacher. A closely defined project may be the best way to learn how to use a computer graphing package or another mathematical tool. In practice, there needs to be a sliding scale of autonomy in project work: an overall plan of increasing autonomy should be operating within the school. Students need to be brought gradually to the point where they can define and plan a complete investigation. Collecting their own data, seeking their own references, planning how to present results, for example, are all decisions which contribute to autonomy and the amount of independence can be manipulated with these factors throughout secondary school.

The arrangement of setting only a theme for the common assessment task enables students to work independently and this is further reinforced by the general advice which encourages them to be independent and to show initiative. The first version of the VCAB project assessment checklist included items about the students' independence in selecting, planning and conducting the investigation. This was abandoned because the teacher's judgement could not be verified and independence cannot be ascertained from the project reports standing alone. The intention that the work be the students' own still remains, but is currently only in the general advice offered to students and not now reflected in the assessment. Allowing students to choose their own topics within the tight timeframe of the CAT means that they have to choose a topic decisively and get on with the job - ditherers and perfectionists beware!

Authenticity

Public credibility of a Year 12 assessment system depends on a confidence that project work done outside examination conditions is in fact the work of the student. This issue, according to Symons' report (1989) of interviews with 1989 trial teachers, was the one with which teachers had the most difficulty. Teachers are required to attest that the work is that of the student and that all assistance has been acknowledged. In order to assist teachers in this, the instructions for undertaking the CAT project have attempted to put the onus on the student to demonstrate involvement to the teacher, rather than have the onus on the teacher to seek evidence of the students' involvement. The guidelines for 1990 specified three particular points at which the student must confer with the teacher; at the choice of topic, the plan and the first draft. As it is expected that students will receive some help from various sources, all help has to be acknowledged in the project report, but no definite guidelines on what is or is not reasonable have been established. Typically these acknowledgements in 1989 were to family members who typed and discussed the project, to local businesses which supplied data to be analysed and to librarians.

Differential access to help for students according to their geographical location (near facilities such as libraries) or their family background and resources also concerns teachers.

Conclusion

Despite the many unresolved issues of using projects as a tool of assessment for Year 12, the reaction from both students and trial teachers has generally been very positive. Students on the whole have enjoyed the tasks themselves and generally accepted the challenge to produce good work. Consequently most teachers found the exercise educationally worthwhile and put in the large amount of time required for assessment. The arrangements for the project rely upon a high degree of professionalism from teachers. Its ultimate success as a method of assessment will in large measure reflect the public's image of their professional competence.

References

Stacey, Kaye (1990) Using Projects To Teach and Assess Senior Mathematics. In Ken Milton and Hugo McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (pp 512 - 518). Hobart: Mathematical Association of Tasmania.

Symons, Michael, R. (1989) *An Evaluation of the Common Assessment Tasks Trials for Mathematics*. Unpublished.

Victorian Curriculum and Assessment Board. (1989 a). *Mathematics course development support material*. Melbourne: author as publisher.

Victorian Curriculum and Assessment Board. (1989b). *Mathematics Study Design*. Melbourne: author as publisher.

Victorian Curriculum and Assessment Board. (1990). VCE Assessment Evaluation Team Report - Summary of Conclusions. *Board Report* , no 43, 11th April 1990.

